Online Appendix
Optimal Redistributive Policy in Debt Constrained Economies

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A Sovereign game

Before setting up the game, consider the general environment in which the government's policy includes the decision to default on debt $\{d_t\}_{t=0}^{\infty}$, where $d_t \in \{0,1\}$ and $d_t = 0$ implies default.[1] The government's budget constraint becomes

$$G_t + (1+r_t)B_t^g \leq \tau_t^n w_t L_t + \tau_t^k r_t K_t + \tau_t^d \left(r_t A_t + \left[(1-\tau_t^k) r_t^k - \delta \right] K_t \right) + d_t B_{t+1}^g + T_t.$$

As the government cannot commit to any of its policies, one can think that the government, domestic agents, and international lenders enter into a sovereign game in which they determine their actions sequentially. In every period, the state variable for the game is $\left\{ B_t^g, \left(k_t^i, a_t^i\right)_{i\in I} \right\}$. The timing of the actions is as follows:

- Government chooses $z_t^G = \left(\tau_t^n, \tau_t^d, \tau_t^k, T_t, d_t, B_{t+1}^g\right) \in \Pi$ such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c_t^i, l_t^i, k_{t+1}^i, a_{t+1}^i\right)$ subject to their budget constraints, the representative firm produces output by choosing $z_t^F = (K_t, L_t)$, and the international lenders choose holdings of bonds $z_t^* = \left(B_{t+1}^g, A_{t+1}\right)$ given the interest rates r_t^*.

Define $h^t = \left(h^{t-1}, z_t^G, \left(z_t^{H,i}\right)_{i\in I}, z_t^F, z_t^*, p\right) \in H^t$ as the history at the end of period t. Note that the history incorporates the government's policy, allocation, and prices. Define $h_p^t = \left(h^{t-1}, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy

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[1] Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its international debt ($d_t = 1$).

is $\sigma_t^G: \ H^{t-1} \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i}: \ H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has the strategy $\sigma_t^F: \ H_p^t \to \mathbb{R}_+^2$, and the international lenders have the strategy $\sigma_t^*: H_p^t \to \mathbb{R}_+^2$. Prices are determined by the pricing rule: $p: \ H_p^t \to \mathbb{R}_+$.

Definition 1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^{t-1}, the policy z_t^G induced by the government strategy maximizes the weighted utility by $\boldsymbol{\lambda}$ subject to the government's budget constraint; (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^\infty$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^\infty$, and prices $\{r_t^*\}_{t=0}^\infty$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibria in which deviation triggers the worst payoff. In this case, the value of deviation is the worst equilibrium payoff.

Proposition 1 (Sustainable equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be part of sustainable equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy; and (ii) for any* t*, there exists* $\underline{U}_t(\cdot)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{s=t}^{\infty} \beta^{s-t} \sum_{i\in I} \lambda^i \pi^i U^i\left(c_s^i, l_s^i\right) \geq \underline{U}_t(K_t).$$

Proof. Define $\underline{U}_t(K_t)$ as the maximum discounted weighted utility for the domestic agents in period t when the government deviates. In period t, the domestic agents and the government can borrow abroad. In subsequent period $s > t$, the economy reverts to the worst equilibrium.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the sustainable equilibrium. Then by the optimal problems of the government, domestic agents, and international lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies the government budget constraint, and satisfies the international lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is a competitive equilibrium with policies. For any period t and history h^{t-1}, an equilibrium strategy that has the government deviate in period t triggers reverting to the worst equilibrium in period $s > t$. Such a strategy must deliver a weighted utility value that is at least as high as the right-hand side of (1). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^{t-1} be any history such that there is no deviation from z^G up until period t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U_t^d(K_t)$ in period t and U^{aut} for period $s > t$. Because the plan is constructed to maximize period-t utility at K_t, the right-hand side of (1) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. □

B Numerical appendix

This section explains the calculation of deviation utility and the computational algorithm solving the model.

B.1 Deviation utility

The deviation utility $\underline{U}$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes where there is no access to international financial markets. Given that output is equal to the total efficiency-unit labor supply, one has

$$\begin{aligned} \underline{U} \equiv \max_{c_t^i, l_t^i, \tau_t^n, T_t} & \sum_{t=0}^{\infty} \beta^t \sum_{i \in I} \lambda^i \pi^i U^i(c_t^i, l_t^i) \\ s.t. \quad & c_t^i + b_{t+1}^{i,d} = (1 - \tau_t^n) l_t^i - T_t + (1 + r_t) b_t^{i,d} \\ & G_t + (1 + r_t) B_t^d \leq \tau_t^n L_t + T_t + B_{t+1}^d \end{aligned}$$

There exist a vector of market weights $\hat{\boldsymbol{\varphi}}$ such that

$$\begin{aligned} \underline{U} \equiv \max_{C_t, L_t, \hat{\boldsymbol{\varphi}}, T} & \sum_{t=0}^{\infty} \beta^t \left[\hat{\Phi}_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \hat{\Phi}_L^W \frac{L_t^{1+\nu}}{1+\nu} \right] \\ s.t. \quad & C_t + G_t \leq L_t \end{aligned}$$

where $\hat{\psi}_c^i, \hat{\psi}_l^i, \hat{\Phi}_C^V, \hat{\Phi}_L^V, \hat{\Phi}_C^W, \hat{\Phi}_L^W$ are calculated using $\hat{\boldsymbol{\varphi}}$.

B.2 Computational algorithm

State variables: μ, Γ

1. Guess μ and φ. Compute η.
   (a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
   Initial guess of the expectation $V(\mu_t, \Gamma_{t-1}) = \sum_{s=t}^{\infty} \beta^{\tau - t} \left[\Phi_C^P \frac{C_s{}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_s{}^{1+\nu}}{1+\nu} \right]$.
   (b) Assume the constraint does not bind in t: $\gamma_t = 0$. Solve for the allocation C_t, L_t using the following first-order conditions

$$\begin{aligned} \left[\mu_t \Phi_C^W + \Phi_C^m \Gamma_{t-1} \right] C_t{}^{-\sigma} = \mu \\ \left[\mu_t \Phi_L^W + \Phi_L^m \Gamma_{t-1} \right] L_t{}^{\nu} = \mu \end{aligned}$$

(c) Compute $V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$, then compute

$$\begin{aligned} A_t &= \sum_{s=t}^{\infty} \beta^{\tau - t} \left[\Phi_C^P \frac{{C_s}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_s}^{1+\nu}}{1+\nu} \right] \\ &= \Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1}, \Gamma_t) \end{aligned}$$

(d) Check if $A_t \geq \underline{U}_t$. If it is, proceed to the next step. If not, solve for C_t, L_t, γ_t using these optimality equations

$$\begin{aligned} \left[\mu_t \Phi_C^W + \Phi_C^m \Gamma_{t-1}\right] {C_t}^{-\sigma} &= \mu \\ \left[\mu_t \Phi_L^W + \Phi_L^m \Gamma_{t-1}\right] {L_t}^{\nu} &= \mu \\ \Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t)) &= \underline{U}_t \end{aligned}$$

(e) Given C_t, L_t, γ_t (γ_t can be zero or not), compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t))$. Update the value function

$$V^{n+1}(s_t, \Gamma_{t-1}) = \Phi_C^P \frac{{C_t}^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{{L_t}^{1+\nu}}{1+\nu} + \beta V^n(\mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* (\Gamma_{t-1} + \gamma_t))$$

2. Compute residuals to find μ and φ

$$\begin{aligned} r^\mu &= \sum_{t=0}^{\infty} q_t \left[L_t - G_t - C_t\right] - B_0 \\ r_{ij}^\varphi &= \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^m \left(\psi_c^i - \psi_c^j\right) {C_t}^{1-\sigma} - \Phi_L^m \left(\psi_l^i - \psi_l^j\right) {L_t}^{1+\nu} \right] \end{aligned}$$

$$r = (r^\mu)^2 + \sum_{i,j} (r_{ij}^\varphi)^2 \tag{B.1}$$

3. Find μ and $\boldsymbol{\varphi}$ such that r is minimized using a Nelder-Mead algorithm.