Sovereign Debt Sustainability and Redistribution*

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Abstract

This paper proposes a theory of sovereign debt sustainability based on the government's motive for redistribution. I study debt and distortionary tax policies in a heterogeneous-agent small open economy with the government's lack of commitment. Access to external financing reduces the distortionary cost of redistribution. Therefore, default resulting in financial autarky is endogenously costly. The theory is quantitatively consistent with Italy's recent external debt buildup and the cross-country correlation between inequality and external debt. Optimal austerity policies are increasing external borrowing and redistribution in the short run, while reducing redistribution to repay debt in the long run.

Keywords: Inequality; Limited commitment; Optimal taxation; Redistribution; Sovereign debt

JEL Classifications: E62; F38; F41; H21; H23; H63

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Introduction

The recent European debt crises have prompted intense policy debates on the design of fiscal policies during severe economic downturns, in which output is low and external debt rapidly increases until it is constrained by the country's inability to repay. Austerity policies such as increasing tax revenue or reducing government expenditure provide more resources to repay debt but have unequal effects on domestic residents.[1] Therefore, appropriate policy design requires understanding the interaction between a government's commitment to debt repayment and its commitment to maintain a level of redistribution. Two key questions arise: How does a government's redistributive goals affect its incentive to repay debt? Given the answer to the first question, how does one design optimal austerity policies taking their distributional consequences into account?

To address these questions, I study a small open economy model in which redistribution comes with an efficiency cost in terms of labor tax distortion and the government faces endogenous borrowing constraints due to its lack of commitment. Given these constraints, the cost of financial autarky endogenously determines the sustainable level of external debt. I examine the trade off between the government's desire for redistribution and its ability to sustain external debt and evaluate optimal responses of fiscal policies to aggregate shocks in the presence of inequality.

I develop a theory of external debt sustainability based on the government's motive for redistribution. When the government can borrow externally, it redistributes the external funds to domestic households via domestic financial markets and levy low distortionary labor taxes. Default resulting in financial autarky is endogenously costly because, without external borrowing, redistribution requires high distortionary taxes, which reduce the economy's efficiency. Therefore, the government is willing to repay its external obligations in order to continue having access to external financial markets. I then show that this theory can quantitatively account for the recent buildup of external debt in Italy and is consistent with the positive correlation between pre-tax income inequality and external debt across countries and over time. The optimal austerity policies responding to a negative productivity shock are increasing external borrowing,

[1] The United Kingdom and Ireland implemented expenditure cuts, while Greece, Italy, Portugal, and Spain implemented both tax increases and expenditure cuts for their austerity plans. Most of these plans include cuts in public services, pension programs, and education programs. Monastiriotis (2011) argues that in Greece, the prolonged fiscal consolidation has exacerbated regional disparities and imbalances. Leventi and Matsaganis (2016) use a micro-simulation model to assess the distributional effects of austerity policies and find that such policies have led to higher poverty and after-tax income inequality, worsening the adverse distributional effects of the recession. Brinca, Homem Ferreira, Franco, Holter, and Malafry (2019) show that fiscal consolidations are more recessive when income inequality is higher.

decreasing average taxes, and increasing redistribution while raising average taxes and reducing redistribution to repay debt in the future.

The model features a continuum of domestic agents that are impatient and differ by labor productivity types. The aggregate shocks are in aggregate productivity and government spending.[2] Domestic and external credit markets consist of state-contingent assets. The tax system has lump-sum taxes as well as marginal taxes that are distortionary to individual labor supply and saving decisions. The government cares about all domestic agents, assigning individual welfare weights that represent its distributional preference. The government lacks commitment in all tax and debt policies.

Concerns for redistribution rationalize the need for distortionary taxation. Since all domestic agents face the same tax rates, the government redistributes resources by levying a positive labor tax alongside a lump-sum transfer. In this way, highly-skilled, high-income agents bear a larger tax burden than low-skilled, low-income ones. Alternatively, the government can use a tax on domestic borrowing and a lump-sum transfer, which implies that the highly indebted agents will pay more taxes than the less-indebted agents. In this environment, levels of tax distortions represent the cost of redistribution.[3]

I consider the sustainable equilibrium that is characterized by self-enforcing borrowing constraints due to the government's lack of commitment, as in Aguiar and Amador (2014, 2016). The government sets an ex ante set of policies such that for any given time period and history, if the government deviates from its equilibrium policies, it triggers a punishment to financial autarky, in which the government is permanently excluded from external credit markets. At the beginning of financial autarky, the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents.[4] The self-enforcing constraints are such that the continuation of value of staying in equilibrium has to be at least the value of financial autarky. These constraints impose endogenous limits on the economy's external borrowing. The equilibrium allocation determines the amount of external debt can that the government can sustain without defaulting.

The main theoretical finding is that the government finds it optimal to sustain posi-

[2] Later on when I match the model to the data, government spending is measured as the government's final consumption of public goods, excluding spending on social welfare programs.

[3] This is because the government can raise lump-sum taxes to finance expenditures and debt repayment without distorting the domestic agents' decisions. Werning (2007) provides a similar intuition. The presence of lump-sum taxes removes the revenue purposes of distortionary taxation.

[4] This is equivalent to the assumption of nondiscriminatory defaults on domestic or external lenders. See, for example, D'Erasmo and Mendoza (2016) for a similar assumption. For an example on discriminatory defaults, see Gonzalez-Aguado (2018).

tive external debt in equilibrium. This result holds given that the government is inequality averse, initial wealth inequality is positively correlated with skill inequality, and the intratemporal elasticity of substitution is at least above the $\log$ preference, and the elasticity of labor supply is at most one. Debt sustainability is optimal because it is costly for the government to default and faces financial autarky.[5] Default is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature, in contrast to the standard exogenous cost of default in terms of output or productivity losses.

Having access to external financing allows the government to redistribute more than in financial autarky. In the contract, future declines in labor taxes improve the economy's efficiency by encouraging higher output and allowing the government to borrow more in the present. This additional unit of resource implies higher present consumption and so higher welfare than financial autarky. Furthermore, the contract exhibits a lower level of redistribution than in financial autarky at the same amount tax distortion in the long run.

The theory is quantitatively consistent with salient features of the data. Using Italy's data, I calibrate the model to match key macroeconomic statistics and average cross-sectional wage inequality and find that the model accounts for the average level and upward trend of the external debt-to-output ratio in Italy for the period 2002-2015, while also being consistent with key business cycle statistics. Moreover, a counterfactual exercise for Italy during the periods 1985-2001 and 2002-2015 exhibits that an increase in the underlying wage inequality that matches the increase in income inequality can account for 71% of the increase in the average debt-to-output ratio. This result is consistent with the theory's prediction that a higher level of inequality, or a higher redistributive motive, corresponds to a higher cost of default, which results in a higher sustainable debt level.

I then study optimal austerity in the presence of inequality through the lens of optimal policy response to shocks. Following a negative innovation of the productivity shock, external debt increases while utility differences among agents decline initially and increase in subsequent periods. More external borrowing allows higher transfers to individuals and more redistribution, while higher taxes and lower redistribution are needed in the future to repay debt. In contrast, the representative agent model only

[5] In some works in the literature, default means not repaying debt. In this model, I use the term default as the case in which the government does not honor any terms of the contract, either in debt repayment or taxes.

borrows a small amount in response to the negative productivity shock.

Endogenous borrowing constraints are essential to the theory because they link the government's external borrowing decisions to its concern for redistribution and tax distortions through the value of financial autarky. If the borrowing constraint is exogenous, given impatience, the only optimal policy is to borrow up to the exogenous debt limit, regardless of the desired level of redistribution or inequality.

The model builds on state-contingent financial markets and features no equilibrium default, in contrast to the sovereign default model.[6] However, the threat of default still affects the optimal allocation, as in Thomas and Worrall (1988) and Kehoe and Levine (1993). The endogenous borrowing constraints imply that there is imperfect insurance against aggregate risk. These constraints also endogenously determine the optimal debt portfolio, in contrast to the incomplete framework in which the the risk-free bond is the only financial asset. Furthermore, defaults in reality often accompany a non-zero net capital flow as a country often goes through a lengthy process of renegotiation and haircuts.[7] This framework embeds part of the default procedure into self-enforcing borrowing constraints, instead of assuming zero net capital flows, as in the standard sovereign default model.[8]

Heterogeneity and distortionary taxation significantly affect the equilibrium sustainable level of external debt, whereas the level of government expenditure have only a minor impact. Intuitively, the value of financial autarky decreases in the presence of distortionary taxes and inequality, which in turn increases the government's incentive to repay external debt.

Lastly, I examine the cross-country empirical support for the theory. In a cross-country panel datasets on inequality and balance of payments, I find that highly indebted countries have also experienced high levels of pre-tax income inequality, which is consistent with the theory's prediction.

Related literature. This paper builds on the sovereign debt literature of limited commitment and state-contingent asset market that follows Aguiar and Amador (2011, 2014). I present the quantitative predictions of this type of model, including highly volatile consumption and fiscal policies, which are similar with the findings in Kehoe

[6] See, for example, Eaton and Gersovitz (1981).

[7] Standard & Poor's defines default as the failure to meet a principal or interest payment on the due date contained in the original terms of a debt issue. This definition covers both missed payments (breach of contract) and distressed debt restructurings that involve losses for creditors. This is the standard default definition used in the literature (e.g., Reinhart and Rogoff (2009)). See Ams, Baqir, Gelpern, and Trebesch (2018) for a discussion on the pros and cons of different definitions of default.

[8] See Restrepo-Echavarria (2019) for a discussion on these issues.

and Perri (2002) for a two-country international real business cycle and Bauducco and Caprioli (2014) with two-sided limited commitment.

I introduce heterogeneity and redistributive effect of fiscal policies in the literature that studies the government's lack of commitment in both tax and debt policies. The volatile tax and government expenditures are similar to Cuadra, Sanchez, and Sapriza (2010). The theoretical finding of declining labor taxes when borrowing is tightened relate to the absence of tax smoothing in Pouzo and Presno (2015) and the quantitative result of Arellano and Bai (2016), in which higher tax distortion would make the country more likely to default. The government's incentive to front-loading tax distortion is also found in Karantounias (2018).

This paper also contributes to the literature of endogenous cost of default beyond insurance motive. In Mendoza and Yue (2012), it is the efficiency loss in production as default prevents the final good producers to finance the purchase of imports, which only have imperfect substitutes at home. Balke (2017) shows how default limits the supply of bank's loans that firms use to finance vacancies and wages. Therefore, default leads to a large increase in unemployment, which is endogenously costly. In this paper, distortionary taxation plays an important role in determining the cost of default. Default is costly because of the high and volatile labor distortions need to redistribute in financial autarky.

This paper finds optimal policy by characterizing the best allocation of any tax-distorted equilibrium, i.e. the primal approach as in the public finance literature (Barro (1979),Lucas and Stokey (1983) Chari, Christiano, and Kehoe (1994), Aiyagari and McGrattan (1998), Chari and Kehoe (1999), Aiyagari, Marcet, Sargent, and Seppälä (2002), and many other papers). The argument for labor tax smoothing in these papers relies on the fact that the government can issue debt that is contingent to all states and is not constrained (in a sense of beyond the natural debt limit). In this paper, tax smoothing is not always optimal; the government's ability to smooth tax distortion is restricted by the willingness to lend by the international lenders.

This paper relaxes the assumption on the government's commitment to policies in many papers that study trade-off between redistribution and debt management. I build upon the framework of optimal taxation with redistribution in Werning (2007), in which perfect tax smoothing occurs under the same assumptions. I establish that binding borrowing constraint due to lack of commitment alter the tax dynamics, resulting in imperfect tax smoothing. In addition, I show how the quantitatively high cost of default of redistribution lead to the high positive debt level in the long run, in contrast to Bhandari, Evans, Golosov, and Sargent (2016)'s finding that the average long-run on optimal public debt is not positive. Similar to Bhandari, Evans, Golosov, and Sargent (2017) which emphasize the importance of the distribution of initial asset holdings, I find that it affects the long-run debt repayment capacity via the equilibrium level of redistribution.

Several recent papers addressed the trade-off between redistribution and external debt. This paper extends the lack of commitment to both tax and debt policies, in contrast to Ferriere (2015) that assumes one-period commitment to tax progressivity, and finds a related result that a more progressive economy finds a higher cost of default because redistribution is more distortionary in autarky. I focus on the long-run trade-off between inequality and external debt, in contrast to Dovis, Golosov, and Shourideh (2016) that argued how this trade-off endogenized the dynamic cycles of fiscal policies over time. My model features redistributive consequences of domestic defaults, as emphasized by D'Erasmo and Mendoza (2016, 2020). I incorporate this insight into linking redistributive incentives and the cost of default, which in turn affects the sustainable level of external debt. Balke and Ravn (2016) studies tax and debt policies from a Markov perfect equilibrium with inequality through unemployment. This paper allows for a more general framework of inequality and redistributive motive and focuses on the ex-ante welfare maximizing policies. My theoretical finding is consistent with their quantitative one, in which it's optimal to minimize tax distortions during crises.

Other papers have documented a positive relationship between income inequality and sovereign debt. Berg and Sachs (1988) show that income inequality is a key predictor of a country's probability of rescheduling debt and the bond spread in secondary markets. Aizenman and Jinjarak (2012) describe a negative correlation between income dispersion and the tax base and a positive correlation with sovereign debt. Jeon and Kabukcuoglu (2018) and Ferriere (2015) also provide evidence that rising income dispersion significantly increases sovereign default risk. This paper provides a theory that can collectively account for the increase in debt and income dispersion.

Outline. The paper is organized as follows. Section 1 describes the environment and sets up the competitive equilibrium. Section 2 formulates the planning problem and the main theoretical results of optimal debt sustainability. Section 3 provides a quantitative analysis and analyzes optimal austerity policies. Section 4 provides a sensitivity analysis, and Section 6 concludes.

1 A Model of Sovereign Debt and Inequality

In this section, I setup the model of a small open economy with aggregate uncertainty, heterogeneous agents, and a benevolent government. I define the competitive equilibrium given government policies and show that the competitive equilibrium can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility shares.

1.1 Environment

A small open economy faces publicly observed aggregate shocks $s_t \in S$ in period t, where S is some finite set. Let $\Pr(s^t)$ denote the probability of any history $s^t = (s_0, s_1, ..., s_t)$, where $\Pr(s^{t+j}|s^t)$ denotes the probability conditional on history s^t, $j \geq 0$. Similarly, $\Pr(s_{t+1}|s^t)$ is the probability period $t+1$'s state is s_{t+1}, conditional on history s^t. When it does not cause confusion, I use x_t to denote ta random variable with a time t for all s^t.

There is a measure-one continuum of infinitely-lived agents different by labor productivity types $(\theta^i)_{i\in I}$, which are publicly observable. The fraction of agents with productivity θ^i is π^i, where $(\pi^i)_{i\in I}$ and $(\theta^i)_{i\in I}$ are normalized such that $\sum_{i\in I}\pi^i = 1$ and $\sum_{i\in I}\pi^i\theta^i = 1$. All agents have the same discount factor β and the static utility $U(c,n)$ over consumption c and hours worked n. The utility of agent with productivity θ^i over consumption $c_t^i \geq 0$ and efficiency-unit labor $l_t^i \geq 0$ is

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \tag{1}$$

where $\sigma, \nu > 0$.

In addition, there is a representative firm that uses labor to produce a single final good. The production function in period t with history s^t is $F(L, s^t, t)$, constant returns to scale, where L is aggregate labor (in efficiency units). The economy is subject to an exogenous sequence of government spending $\{G_t\}_{t=0,s^t\in S^t}^{\infty}$. Both the production function and government expenditures depend on the time period t, capturing deterministic changes such as growth, and the history s^t, capturing the uncertainty impact.

An individual allocation specifies consumption and labor in every period after every history for each agent i: $\{(c_t^i, l_t^i)\}_t$. The aggregates are denoted by $C_t \equiv \sum_{i\in I}\pi^i c_t^i$ and $L_t \equiv \sum_{i\in I}\pi^i l_t^i$.

The exogenous risk-free international interest rate for borrowing is r^*. Both the domestic and international financial markets are competitive. The government can issue domestic debt from a full set of state-contingent bonds, which can be traded across agents. The government also have access to a full set of state-contingent external bonds. Let $R^* = 1 + r^*$ denote the gross risk-free interest rate. Define $Q_t(s_{t+1}) = \Pr(s_{t+1}|s^t)/R^*$ as the international price of one unit of consumption at state s_{t+1} in period $t+1$, conditional on history s^t, in units of consumption at history s^t. Similarly, $q_t = \Pr(s^t)/(R^*)^t$ is the international price of one unit of consumption at history s^t in units of consumption at s^0. Let normalize $q_0 = 1$.[9] Note that $q_{t+1} = Q_t(s_{t+1})q_t$. I assume that only the government can borrow abroad.[10]

1.2 Competitive Equilibrium

In every period t and history s^t, the government can issue both domestic and foreign bonds and impose a lump-sum tax T_t, a marginal tax on labor income τ_t^n, and a tax on the return of domestic saving τ_t^d. The firm and agents face the labor wage $w(s^t)$.

Domestic agent. Individual agent of type $i \in I$ faces the sequential budget constraint

$$c_t^i + \sum_{s_{t+1}|s_t} Q_t^d(s_{t+1})b_{t+1}^{d,i} \leq (1-\tau_t^n)w_t l_t^i + (1-\tau_t^d)b_t^{d,i} - T_t, \tag{2}$$

where $c_t^i, l_t^i, b_t^{d,i}$ denote the consumption, labor, and domestic bond holding of agent i in period t and history s^t, respectively. $Q_t^d(s_{t+1})$ is the price of one unit of domestic asset for realization s_{t+1} in period $t+1$ given history s^t.

Representative firm. The firm chooses aggregate labor to maximize profit

$$\max_{\{L_t\}_t} \quad F\left(L_t, s^t, t\right) - w_t L_t,$$

which gives the following first-order condition

$$w_t = F_L\left(L_t, s^t, t\right). \tag{3}$$

[9] This normalization is without loss of generality since the initial level of external debt is fixed.

[10] In the data, domestic residents often hold a very small amount of foreign assets, so most models assume that they do not have access to the external credit market. In this environment, the set up is equivalent to the case where the domestic agents can save abroad with the bond price $Q^*(s^t)$, but then face a residence-based tax $\tau^d(s^t)$. External debt will be the net foreign liability of both the private and public sectors, instead of only the public sector here. I choose this particular set up so that it is more straightforward to characterize the strategic game later on in Appendix B.

The firm's profit is zero in equilibrium because of the constant-return-to-scale production function.

Government. There is an exogenous government expenditure $\{G_t\}_{t=0,s^t\in S^t}^{\infty}$. Given the one-period state-contingent domestic bond B_t^d and external bond B_t, the government's budget constraint is

$$G_t + (1-\tau_t^d)B_t^d + B_t \leq \tau_t^n w_t L_t + \sum_{s_{t+1}|s_t} Q_t^d(s_{t+1})B_{t+1}^d + \sum_{s_{t+1}|s_t} Q_t(s_{t+1})B_{t+1} + T_t,$$

where $B_t^d = \sum_{i\in I}\pi^i b_t^{d,i}$ is aggregate domestic bond, and B_t is the amount of the government's external debt. There is a no-Ponzi condition such that the present value of external debt is bounded below.

The government's present-value budget constraint is

$$\sum_{t\geq 0,s^t} q_t \left\{ G_t - \tau_t^n w_t L_t - T_t + \sum_{s_{t+1}|s_t} Q_t(s_{t+1})B_{t+1}^d - (1-\tau_t^d)B_t^d \right\} \leq B_0. \tag{4}$$

Resource constraint. Using the agent's budget constraints and government's budget constraint, one can obtain a present-value resource constraint in terms of the intertemporal international prices and the initial external debt,

$$\sum_{t\geq 0,s^t} q_t \left[F(L_t, s^t, t) - G_t - C_t \right] \geq B_0. \tag{5}$$

Competitive equilibrium. Given the above equations, one can define the following competitive equilibrium with government policies.

Definition 1.1. Given initial external debt B_0 and individual individual bond positions $\left(b_0^{i,d}\right)_{i\in I}$, a competitive equilibrium with government policies for an open economy is individual agent's allocation $z^{H,i} = \left\{\left(c_t^i, l_t^i, b_t^{i,d}\right)\right\}_{t=0}^{\infty}$, $\forall i \in I$, the representative firm's allocation $z^F = \{L_t\}_{t=0}^{\infty}$, prices $p = \left\{q_t, w_t, Q^d(s_{t+1}|s^t)\right\}_{t=0}^{\infty}$, and government's policy $z^G = \left\{\tau_t^n, \tau_t^d, T_t, B_t^d, B_t\right\}_{t=0}^{\infty}$ such that (i) given p and z^G, $z^{H,i}$ solves individual i's problem that maximizes (1) subject to (2) and a no-Ponzi condition of agent's debt value, (ii) given p and z^G, z^F solves firm's problem, (iii) the government budget constraint (4) holds, (iv) the aggregate resource constraint (5) is satisfied, (iv) the domestic bond market clears $B_t^d = \sum_{i\in I}\pi^i b_t^{d,i}$, and (v) p satisfies $q_t = \Pr(s^t)/(R^*)^t$ and equation (3) given z^G.

1.3 Characterizing Competitive Equilibrium

In equilibrium, the intra-temporal and inter-temporal rates of substitution are the same across agents, i.e. in each period t and each history s^t, for any individual i,

$$(1-\tau_t^n)w_t = -\frac{1}{\theta^i}\frac{(l_t^i/\theta^i)^\nu}{(c_t^i)^{-\sigma}},$$
$$\frac{Q_t^d(s_{t+1})}{1-\tau_t^d} = \beta\Pr(s_{t+1}|s^t)\frac{\left(c_{t+1}^i\right)^{-\sigma}}{(c_t^i)^{-\sigma}}.$$

Given the aggregate allocation (C_t, L_t), there is an time-invariant efficient assignment of individual allocation $(c_t^i, l_t^i)_{i\in I}$ due to the equal marginal rates of substitution between consumption and labor. Any inefficiencies due to tax distortions are captured by the aggregate allocation. This property allows the competitive equilibrium allocation to be characterized in terms of aggregates and a static rule for individual allocation.

For any equilibrium, there exist a set of Neghishi weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$, with $\varphi^i \geq 0$ and $\sum_i \pi^i\varphi^i = 1$, such that individual allocation solve a static problem

$$V(C,L;\boldsymbol{\varphi}) \equiv \max_{(c^i,l^i)_{i\in I}} \sum_{i\in I}\varphi^i\pi^i\left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega\frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu}\right]$$
$$s.t. \qquad \sum_{i\in I}\pi^i c^i = C; \quad \sum_{i\in I}\pi^i l^i = L$$

This problem gives the allocation rule for individual i that is time-invariant prorportional to the aggregate allocation

$$c_t^i = \psi_c^i C_t, \quad l_t^i = \psi_l^i L_t, \tag{6}$$

where

$$\psi_c^i = \frac{(\varphi^i)^{1/\sigma}}{\sum_{i\in I}\pi^i(\varphi^i)^{1/\sigma}}, \quad \psi_l^i = \frac{(\theta^i)^{\frac{1+\nu}{\nu}}(\varphi^i)^{-1/\nu}}{\sum_{i\in I}\pi^i(\theta^i)^{\frac{1+\nu}{\nu}}(\varphi^i)^{-1/\nu}}. \tag{7}$$

In addition, V inherits the separable and isoelastic properties,

$$V(C_t,L_t;\boldsymbol{\varphi}) = \Phi_C^V\frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^V\frac{L_t^{1+\nu}}{1+\nu},$$

where Φ_C^V, Φ_L^V depend on $\boldsymbol{\varphi}$. (see Appendix D.1). The envelope conditions of the static problem give

$$(1-\tau_t^n)w_t = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}}, \tag{8}$$

$$\frac{Q_t^d(s_{t+1})}{1-\tau_t^d} = \beta \Pr(s_{t+1}|s^t)\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}. \tag{9}$$

Furthermore, the present-value budget constraint for individual i can be written as

$$\mathbb{E}_0 \sum_{t\geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu}\right) = \Phi_C^V C_0^{-\sigma}\left(b_0^i - T\right), \tag{10}$$

where T is the present-value of lump-sum taxes.[11] Equation (10) is the individual implementability constraint.

Then I have the following proposition in which the competitive equilibrium can be characterized by a set of aggregate allocation and a time-invariant distribution of marginal utility shares.

Proposition 1.1. *Given the initial external debt B_0 and individual bond holdings$\{b_0^i\}_{i\in I}$,an allocation $\{C_t, L_t\}_{t=0}^\infty$ can be supported as an aggregate allocation of an open economy competitive equilibrium with taxes if and only if the resource constraint (5) holds, and there exist market weights $\boldsymbol{\varphi} = (\varphi^i)_{i\in I}$ and lump-sum tax T such that the implementability constraint (10) holds for all $i \in I$.*

Proof. See Appendix. □

2 Optimal Debt Sustainability

2.1 Sustainable Equilibrium

I define the problem of a benevolent government that cares about redistribution but lacks commitment in both tax and debt policies. The solution to this problem is the sustainable allocation in which the government does not default on its debt.

Given a set of social welfare weights $\boldsymbol{\lambda} = (\lambda^i)_{i\in I}$, the government's objective is the weighted utility of all domestic agents

$$\sum_{i\in I} \lambda^i \pi^i \mathbb{E}_0 \sum_{t=0}^\infty \beta^t \left[\frac{(c_t^i)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu}\right]. \tag{11}$$

[11] $T \equiv \sum_{t=0}^\infty \beta^t \sum_{s^t\in S^t} \frac{V_C\left[h^i(C(s^t),L(s^t);\boldsymbol{\varphi})\right]}{V_C\left[h^i(C(s_0),L(s_0);\boldsymbol{\varphi})\right]} T(s^t)$

However, in every period and history node, the government cannot commit to future choices on repayments of debt and taxes. Following Chari, and Kehoe (1990, 1993), the policies are determined in a repeated game between the government, a continuum of domestic agents, and a continuum of international creditors. The sustainable equilibrium supported by trigger strategies to autarky is characterized by the competitive equilibrium conditions described in Proposition 1.1 and the following self-enforcing constraint

$$\sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} \left[\frac{\left(c_k^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_k^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right] \geq \underline{U}_t(s^t, t), \ \forall t, \forall s^t, \tag{12}$$

where $\underline{U}(s^t, t)$ is the one-shot deviation value in which the government defaults on both domestic and external debt and fully redistributes wealth among domestic agents.[12] The government is then in financial autarky, in which it it has no access to external financial markets. $\underline{U}(s^t, t)$ is the value associated with an allocation of a closed economy where the initial states are realized s_t at period t and history s^t, the initial wealth inequality among agents are equal, and the net supplies of domestic and international bonds are zero.

The self-enforcing constraint captures the time-inconsistency of the government policies. If there is a positive net external debt, the government has an incentive to default externally to increase domestic consumption and leisure. In addition, in every history node, there is a non-degenerate distribution of wealth across the domestic agents. The inequality-averse government will also have an incentive to expropriate all the wealth and equally redistribute it. The self-enforcing constraint imposes a limit on the utility, which endogenously determines a limit on external debt for every period and history. These constraints act as endogenous borrowing constraints.

Given the above set-up, a susainable allocation is defined as follows

Definition 2.1. A sustainable allocation $(\{C_t, L_t\}_{t=0}^{\infty}, \boldsymbol{\varphi})$ maximizes the social welfare function (11) and satisfies the conditions in Proposition 1.1 and the self-enforcing borrowing constraint (12)

The objective of the government can be rewritten in terms of aggregate allocation and the Negishi weights

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right),$$

[12] See Appendix B for the formal set up of the sovereign game and its equilibrium characterization.

where Φ_C^P, Φ_L^P are functions of $\boldsymbol{\lambda}$ and $\boldsymbol{\varphi}$ (see Appendix D.1).

The sustainable allocation is part of the solution to the following planning problem

$$(P) \equiv \max_{\{C_t, L_t\}_t, \boldsymbol{\varphi}, T} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\Phi_C^P \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_t^{1+\nu}}{1+\nu} \right)$$

$$\begin{aligned} s.t. \qquad & \sum_{t\geq 0, s^t} q_t \left[F(L_t, s^t, t) - G_t - C_t \right] \geq B_0 \\ & \forall i,\ \mathbb{E}_0 \sum_{t \geq 0} \beta^t \left(\Phi_C^V \psi_c^i C_t^{1-\sigma} - \Phi_L^V \psi_l^i L_t^{1+\nu} \right) = \Phi_C^V C_0^{-\sigma} \left(b_0^i - T \right) \\ \forall t, \forall s^t,\ & \sum_{i \in I} \lambda^i \pi^i \mathbb{E}_t \sum_{k \geq t} \beta^{k-t} \left[\Phi_C^P \frac{C_k^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L_k^{1+\nu}}{1+\nu} \right] \geq \underline{U}_t(s^t, t). \end{aligned}$$

The first constraint is the resource constraint. The second constraint is the implementability constraints that take into account the distortionary effect of the government's policies on individual decisions. The last constraint is the borrowing constraint due to the government's lack of commitment. Domestic agents do not directly internalize the effect of their borrowing decisions on these borrowing constraints. The government, on the other hand, has to consider these constraints when choosing optimal allocation and policies. Therefore, the borrowing constraints indirectly affect domestic borrowing choices via the government's decision on domestic saving taxes.

Appendix C provides the characterization of the sustainable allocation and optimal tax policies that implement the sustainable allocation in competitive equilibrium.

2.2 Optimal Debt Policy

I now argue that the optimal policy is to sustain positive external debt in equilibrium under the following assumptions

Assumption 1 (Impatience)**.** *There exists* $0 < \mathcal{M} < 1$ *such that* $\beta R^* < \mathcal{M} < 1$.

Assumption 2. *The welfare weights, skill distribution, and initial wealth satisfy the following properties*

1. *Redistributive motive towards the low skills:* $\theta^i < \theta^j \iff \lambda^i > \lambda^j,\ \forall i, j \in I$
2. *Perfect correlation between skill and initial wealth:* $\theta^i < \theta^j \iff b_0^i < b_0^j,\ \forall i, j \in I$
3. *Elasticities of substitution and labor supply are such that* $\sigma \geq 1$ *and* $1/\nu \leq 1$

The first assumption implies that the domestic agents are impatient, and therefore, there is a need for debt accumulation. The country will accumulate debt over time, as the international interest rate is lower than the domestic intertemporal rate. For the second set of assumptions, the first part is on the welfare weights which states that the government has a high motive of redistribution towards the lower skill, lower income individuals. The government is inequality averse. The second part makes sure that there direction of inequality in skill is the same as the direction of inequality in initial wealth, meaning that lower skill individuals start off with lower initial wealth endowment. The last part implies that the intratemporal elasticity of substitution is at least above the $\log$ preference, and the elasticity of labor supply is at most 1. This assumption determines the direction of change in optimal tax and debt policies in response to intratemporal and intertemporal changes.

Given that the government cares about redistribution towards low skilled workers, I argue that it is costly for the government to redistribute in financial autarky. That is,

Proposition 2.1. *Suppose Assumptions 1–2 hold, then the optimal labor tax is positive in financial autarky.*

Proof. See Appendix. □

Because of this cost of efficiency in autarky, the government is willing to sustain external debt rather than defaulting. That is,

Proposition 2.2 (Sustainable Debt)**.** *Suppose Assumptions 1–2 hold, then it is optimal for the government to sustain positive external debt in equilibrium.*

Proof. See Appendix. □

The proof relies on the argument that it is not optimal for the government follows financial autarky. Given that there is positive labor distortion in financial autarky, one can construct a deviation from the financial autarkic allocation in which lowering the labor distortion in future periods without changing the continuation value allows the economy to produce more than its consumption in those periods. This deviation implies that the government can borrow today and repay the debt in the future. The extra unit of borrowing allows for more consumption, and so the deviation gives higher welfare than financial autarky.

2.3 Mechanism

In this framework, the government is willing to sustain external debt in equilibrium instead of defaulting because of two reasons: insurance and redistribution. The former comes from the fact that without external credit markets, the government cannot insure itself against aggregate fluctuations. The insurance incentive is standard in the literature (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), Chatterjee and Eyigungor (2012), and many other papers). In this paper, I argue that the government also wants to sustain external debt because it is less costly to redistribute when the government has access to external credit markets.

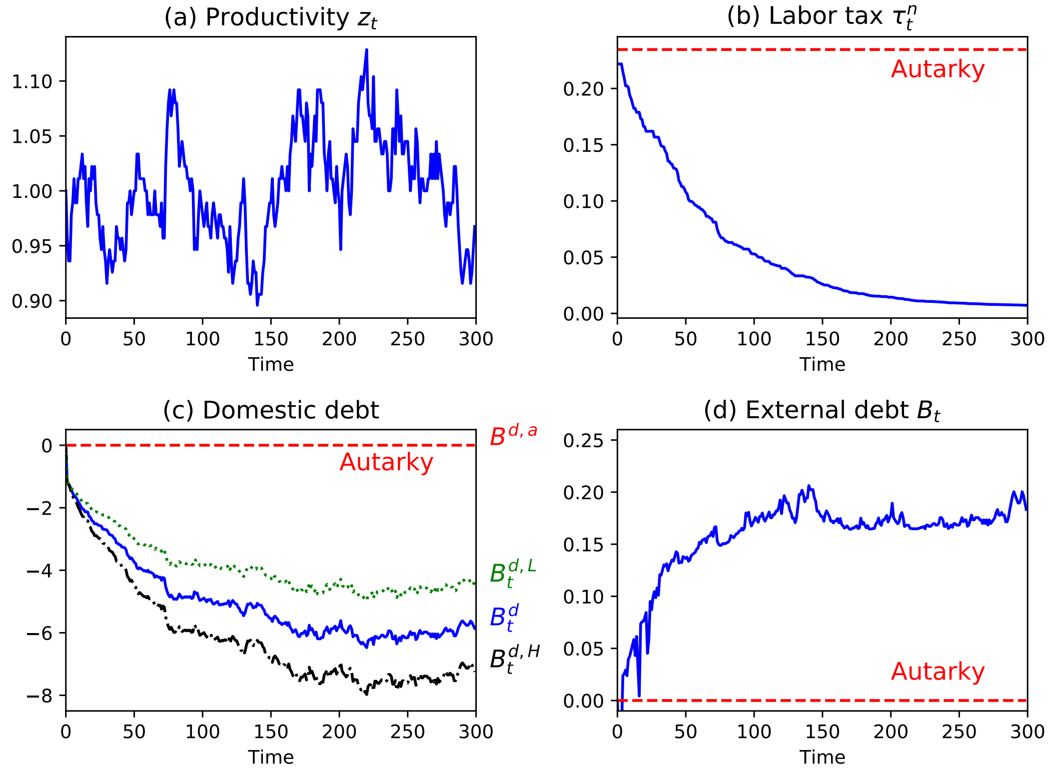
What entails the redistributive benefit of external debt repayment? By repaying debt and having access to external credit markets, the government can redistribute resources via domestic credit markets. This additional redistributive tool allows the government to distort labor supply less. However, if the government defaults and stays in financial autarky, it can only rely on labor taxes for redistribution. As a result, the labor distortion is high in autarky. In other words, external debt repayment allows the government to reduce the efficiency cost of redistribution.

Figure 1 plots simulated time paths of aggregate productivity, labor tax, domestic debt, and external debt in equilibrium and autarky for a two-agent economy.[13] Panel (a) plots the aggregate productivity path, which is the same for both equilibrium and autarky. Panel (b) plots the optimal labor taxes over time. Panel (c) plots the aggregate and individual domestic debts. Note that these are domestic debts issued by the government, so a negative value means that the domestic agents are indebted. Panel (d) plots the external debt levels.

In equilibrium, the high-income agents borrow more over time and become net domestic debtors in the long run. This feature implies that the domestic credit market is a channel for redistributing wealth from high-income agents to low-income agents. As the government borrows more externally, it redistributes resources via a combination of increasing wealth redistribution via domestic credit markets and decreasing labor taxes over time. In contrast, when in autarky, domestic agents net domestic debt holdings are zero. The government can only rely labor taxes for redistribution. The optimal labor tax in autarky is constant at a higher level than the one in equilibrium. In the long run, the government wants to sustain debt because defaulting creates an adverse distributive effect in which all domestic wealth are equalized across agents, so it implicitly transfers more resources to the high income agents.

[13] For details on the underlying assumptions and parameter values, see Section 3.

Figure 1: Simulated time paths of aggregates in equilibrium and autarky



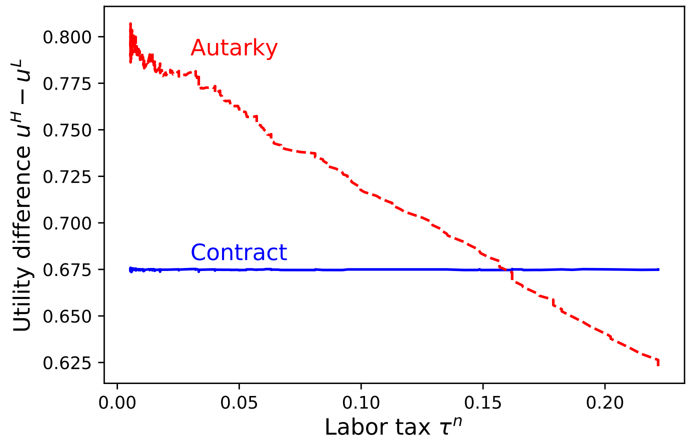
Note: The graph plots the simulated time paths of optimal policies and aggregates of the government's problem in equilibrium and autarky. The implementation is that lump-sum taxes only occur in period 0. Panel (a) and (b) plot the realized aggregate productivity path and the optimal labor tax, respectively. Panel (c) plots the aggregate and individual domestic debt holdings. In autarky, all agents hold zero domestic debt. Panel (d)

Efficiency cost of redistribution in the long run. Figure 2 plots the individual utility difference with respect to a given level of labor tax in the contract and in autarky after the government defaults in period 500. Initially, when labor tax is high, the utility difference in the contract is higher than in autarky. However, in the long run, when the optimal labor tax is low, the utility inequality in the contract is lower than in autarky. Therefore, in the long run, to achieve the same level of redistribution, autarky requires a higher cost of efficiency.

3 Quantitative Analysis

In the previous section, I argue that the government's incentive to sustain debt depends not only on the insurance benefit but also the redistributive benefit of having access to external credit markets. In this section, I quantify these two channels that affect

Figure 2: Labor tax and utility difference



Note: The graph depicts the level of utility difference $u^H - u^L$ for a given level of labor taxes τ^n in the contract and in financial autarky after the government defaults in period 500. The contract line comes from the simulated data for 1500 periods. The autarky line comes from solving the autarkic allocation given each level of labor tax.

debt sustainability using calibrated parameters that match Italian data. I also show that the model is quantitatively consistent with the positive correlation between income inequality and external debt in the cross section and over time as described in the empirical motivation.

3.1 Parameterization

For the quantitative exercise, I assume the following distributional and functional forms. The economy is populated by two types of agents with labor productivity $\{\theta^H, \theta^L\}$, where $\theta^H \geq \theta^L > 0$ and $\pi^H = \pi^L = 0.5$. The planner is utilitarian, i.e. $\lambda^H = \lambda^L$. The individual preference has the form of

$$U^i(c, l) = \log c - \frac{l^{1+\nu}}{1+\nu}$$

The production function is linear in labor, i.e. $F(L, z) = zL$, where z is the aggregate productivity. The aggregate shock is z_t that follows a logged $AR(1)$ process,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_t^z, \ \epsilon_t^z \sim \mathcal{N}(0, \sigma_z),$$

where ρ_z, σ_z are the auto-correlation and the residual standard deviation, respectively. I discretize the productivity process into a Markov chain using Tauchen method with 31

evenly-spaced nodes. From now on, I will use z in place of s as the source of aggregate uncertainty. The government expenditure is constant over time and across histories: $G_t = \bar{g}$. The initial debt levels are $B_0 = 0$ and $b_0^{H,d} = b_0^{L,d} = 0$. The economy starts at the mean of the productivity distribution. The deviation utility $\underline{U}(z^t, t)$ is calculated as the closed-economy version of the model that starts with productivity z_t, zero external debt, and all domestic individuals start with the same initial wealth. $\underline{U}(z^t, t)$ varies with respect to the realized shock z_t.

With these assumptions, the model requires giving values to the parameters of (i) the aggregate productivity process, ρ_z and σ_z; (ii) the cross-sectional wage ratio, θ^H/θ^L; (iii) the individual preference, β and ν; (iv) the government expenditure $\bar{g}$; and (v) the risk-free rate r^*. Table 1 summarizes the parameter values and targets from the calibration exercise.

Table 1: Parameters and Targets

Parameter	Description	Value	Target
Externally calibrated parameters			
r^*	Risk-free rate	0.017	Avg. real return on German bond
β	Discount factor	0.967	Avg. Italian real interest rate = 3.4%
$1/\nu$	Labor elasticity	0.5	Standard literature value
θ^H/θ^L	Wage ratio	1.89	Mean top 50% wage / mean bottom 50% wage
ρ_z	Auto-corr. of prod.	0.927	Auto-corr. of log GDP
Internally calibrated parameters			
σ_z	Std. dev. of prod. res.	0.0205	Std. dev. log GDP
$\bar{g}$	Govt. spending	0.202	Avg. govt. consumption-to-GDP

Note: The table describes the parameters, their values, and the targets in the calibration exercise. Statistics are annual. The risk-free rate and discount factor cover the period of 2002-2015. Wage ratio is the author's calculation from the household-level data set by *Survey on Household Income and Wealth* covering the period of 2002-2014. Auto-correlation and standard deviation of GDP and government final consumption cover the period of 1985-2015. Data sources: *Survey on Household Income and Wealth* (2014), Eurostat (2019), and The World Bank (2019)

A period in the model is one year. For output, I use the logged and linear detrended real GDP series from 1985-2015. I set the auto-correlation of productivity,ρ_z, equals to the auto-correlation of output, which is 0.928. To measure the wage ratio θ^H/θ^L, I use the household-level data from the *Survey on Household Income and Wealth* (SHIW) conducted by the Bank of Italy.[14] Hourly wage is defined as total real compensation of employees, including fringe benefits, divided by total hours worked in a year.[15] For each year in the database, I calculate the ratio of the mean wage of the top 50% of the wage distribution to the mean wage of the bottom 50%. Then θ^H/θ^L is set to 1.89, which is the time-average of these wage ratios for the period of 2002 to 2014. The discount factor β is set to 0.967 so that the average real domestic interest rate is 3.4% for Italy from 2002 to 2015. I choose $\nu = 2$ so that the elasticity of labor supply is 0.5, a standard value in the literature. The risk-free rate is set at 0.017, which is the real rate of return on the German government bonds for the period 2002-2015 (these are secondary market returns, gross of tax, with around 10 years' residual maturity). The interest rate series start at 2002 to isolate the effect of currency and exchange rate risks.[16]

The two remaining parameters, σ_z and $\bar{g}$, are selected to match (i) the standard deviation of logged output and (ii) the government's final consumption-to-GDP ratio for the period 1985-2015. I use the simulated method of moments (SMM). Departing from the quantitative literature on sovereign debt, I do not target the average external debt-to-output ratio but instead leave it as one of the non-targeted moments.[17]

3.2 Moments

Table 2 shows the moments for the data, baseline, and alternative models. The first column reports the statistics from the data for Italy in the period of 1985-2015, except for external debt moments covering the period of 2002-2015. The second column reports the statistics from simulating the model and taking the long-run averages.[18] The calibration successfully matches the standard deviation of output and the government consumption-output ratio for Italy.

External debt. Most importantly, the model is able to produce a quantitatively large amount of external debt-to-output, in consistence with the data. The model generates around 17% of external debt-to-output ratio, average at the ergodic distribution, comparing to 24% of external debt-to-output ratio in the data. The model also matches the volatility of external debt-to-output ratio in the data. These features are with the pres-

[14] See Appendix A.2 for details on the microeconomic data and sample selection.

[15] Real data calculation uses CPI indices provided by the OECD.

[16] See Appendix A for more data descriptions and sources

[17] The following subsection shows the the results of non-targeted moments. Alternatively, the discount factor β can be used to target the debt-to-output ratio.

[18] All model's moments are long-run averages of simulating the economy for 10500 periods and discarding the first 500 periods.

Table 2: Moments: Data, Baseline, and Alternative Models

	Data	Baseline	No inequality	Skill-dependent lump sum tax
Targeted moments				
Std. output (%)	5.3	5.3	5.4	5.4
Avg. govt. expenditure/output (%)	19	19	19	19
Non-targeted moments				
External debt property				
Avg. external debt/output (%)	24	17	2.9	2.9
Std. external debt/output (%)	2.7	2.1	0.45	0.45
Cyclical property				
Std. consumption / Std. output	1.0	1.2	1.2	1.1
Std. net savings/output (%)	1.5	1.8	1.8	1.7
Correlation with output (%)				
Consumption	97	95	95	95
Net savings/output	40	31	36	35

Note: This table reports the non-targeted statistics of the data and the model. The first column reports data statistics which are across the period of 1985 to 2015, except for external debt moments covering the period of 2002 to 2015. The other columns report statistics coming from models' simulations for 10500 periods and excluding the first 500 periods. The no-inequality model corresponds to the case in which $\theta^H = \theta^L$. The skill-dependent-lump-sum-tax model corresponds to the case in which the government has access to skill-dependent lump-sum taxes $(T^i)_{i \in I}$. In the data, government expenditure is final government consumption excluding social transfers. Net saving is defined as output minus total private and government consumption in the data and the model. External debt is defined as the country's net financial liability in the data. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

ence of a relatively high discount factor (0.969) and no additional exogenous cost of default in terms of output/productivity loss.

Cyclical properties. Several cyclical features of the Italian data stand out. First, consumption is as volatile as output and is highly correlated with output. Net saving, defined as output minus the total private and public consumption, only has a volatility of more than a quarter of the volatility of output and a positive correlation with output that is around 40%.[19] The model matches closely the quantitative patterns of the data. The volatility of consumption and net savings relative to output are slightly higher in the model than in the data. Both model consumption and net saving are pro-cyclical with

[19] Neumeyer and Perri (2005) reported key business cycle statistics for both advanced and emerging market economies.

similar correlation levels as in the data. The model is able to generate realistic cyclical patterns of the data, in contrast to the standard model of complete markets. The main reason is that, even with state-contingent assets, the occasionally binding borrowing constraints lead to an imperfect insurance across states and time periods.

3.3 Redistributive Benefit and Distortionary Cost

The redistributive benefit to sustain external debt is quantitatively significant and driven by the distortionary cost of redistribution. To show these features, I compare the baseline model to the two alternative models of no inequality and skill-dependent lump-sum taxes.

The no-inequality model corresponds to the case in which $\theta^H = \theta^L$, and its moments are reported in the third column of Table 2. The no-inequality model generates lower level of external debt, only 2.9% of GDP, comparing to 17% of GDP in the baseline model. This feature implies that the insurance benefit to sustain external debt, in which having access to international financial markets allow the government to insure itself against aggregate uncertainty, only accounts for quantitatively small level of external debt. On the other hand, the redistributive benefit to sustain external debt, in which having access to international financial markets allow the government to redistribute at lower distortionary costs, accounts for quantitatively large level of external debt. The no-inequality model also has lower external debt volatility comparing to the baseline model.

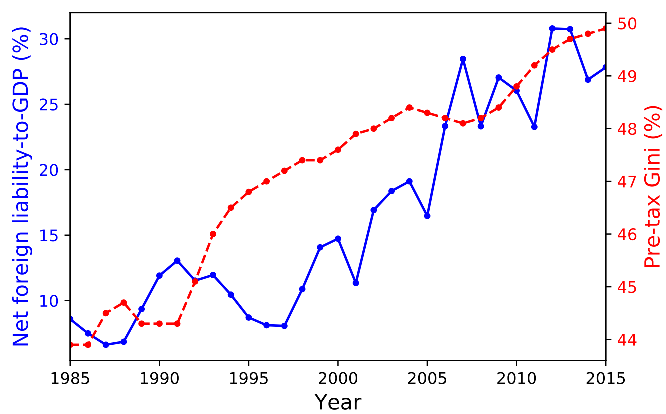
To highlight the role of the distortionary cost of redistribution, I consider the skill-dependent-lump-sum-tax model in which the government has access to lump-sum taxes that are dependent on household's skill levels. In this alternative model, the government can achieve perfect redistribution without levying positive distortionary taxes by taxing or transferring specific amount of resources to each type of households. The fourth column of Table 2 reports the moments of this model. Comparing to the baseline model, the skill-dependent-lump-sum-tax model gives lower level of external debt-to-output ratio and a lower external debt volatility. In skill-dependent-lump-sum-tax model, the government redistributes by lump-sum taxing the highly skilled households and lump-sum transferring resources to lowly skilled households.[20] In the baseline model, the government can only redistribute by levying positive distortionary taxes.

[20]The present value-lump-sum tax-to-output ratio for the highly skilled households is 13, and the present-value-lump-sum-transfer-to-output ratio for the lowly skilled is 15.

3.4 Effect of Inequality on External Debt Over Time

This subsection examines the model's implication on the effect of income inequality on external debt over time. I conduct a comparative statics exercise in the case of Italy for two time periods of 1985-2001 and 2002-2015. Figure 3 plots the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. On average, in the period of 1985-2001, Italy has a lower levels of income inequality and external debt comparing to the period of 2002-2015.

Figure 3: Income inequality and external debt in Italy



Notes: The graph shows the time series of net foreign liablity-to-GDP (%) and pre-tax Gini (%) of Italy from 1985 to 2015. The left y-axis depicts the values in net foreign liablity-to-GDP (%), and the right y-axis depicts the values in pre-tax Gini (%) . Sources: Lane and Milesi-Ferretti (2018), and Solt (2019).

Table 3: Comparative statics results for periods 1985-2001 and 2002-2015

Statistics	Data	Model
Targeted		
Δ Pre-tax Gini	3.0%	3.0%
Non-targeted		
Δ Extenal debt/Y	14%	10%

Notes: The table reports the results of the comparative statics exercise. The first column reports the changes in the data statistics, computed as the average statistics of period 2002-2015 minus the average statistics of period 1985-2001. The second column reports the results from the model. The change in the model statistics is computed as the average statistic of a simulation for the model with the wage ratio equal to 1.89 minus the same statistic of the model with the wage ratio equal to 1.83.

The comparative statics exercise is as follows. I feed into the model a value of wage inequality for the period 1985-2001 and keep other parameter values fixed. I compute ergodic means of pre-tax Gini index and external-debt-to-output ratios. The 1985-2001 value of wage inequality is such that the change in the average pre-tax Gini income from 1985-2001 to 2002-2015 is the same as the change in the data. Table 3 reports the results of the policy experiment. Given the targeted increase in the pre-tax Gini indices in Italy from 1985-2001 to 2002-2015, the model can account for about 71% of the increase in the external debt-to-output ratio.

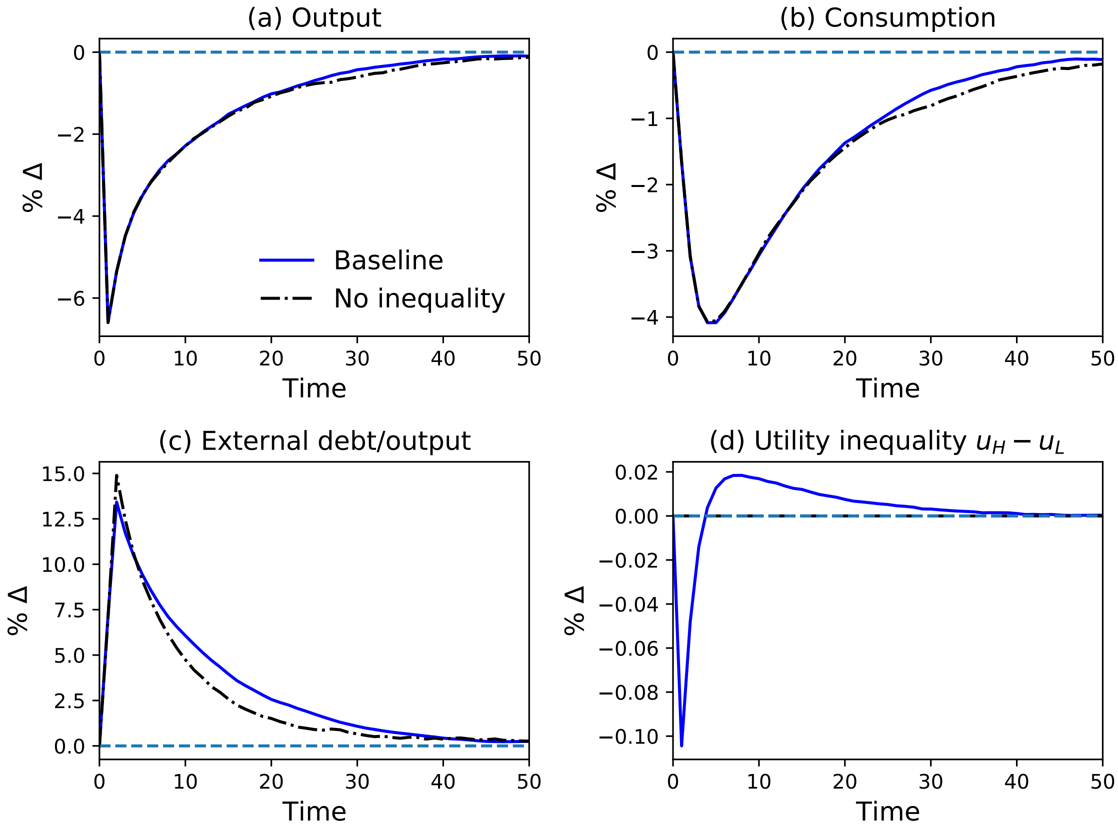
3.5 Optimal Austerity Policies

In this subsection, I study optimal austerity policies in the presence of inequality by measuring the responses of policies to a negative productivity shock. I simulate 30000 paths for the model for 2050 periods. From periods 1 to 2050, the aggregate productivity shock follows its underlying Markov chain so that the cross-sectional distribution of debt converges to the ergodic distribution. In period 2051, normalized to 1 in the plots, the aggregate productivity drops by one standard deviation of the error term. From period 2051 on, the aggregate productivity shock follows the conditional Markov chain. The impulse responses plot the average, across 30000 paths, of the variables from period 2050 to 2055.

Figure 4 plots the impulse responses of output, consumption, external debt-to-output, and utility inequality $u^H - u^L$, for the baseline model and the model of no inequality ($\theta^H = \theta^L$). A decline in productivity growth leads to declines in both output and consumption with a higher drop in output. External debt-to-output increases in response to a low productivity. The utility inequality initial decreases then increases subsequently, implying that there is high redistribution in the short run and low redistribution in the long run. Intuitively, a negative shock leads to a reduction in the deviation utility, and so the borrowing constraint becomes non-binding. The non-binding constraint allows the government to accumulate external debt, temporarily decreases average tax rates, and increases redistribution. In future periods, the government raises taxes to repay the debt and reduces redistribution.

The no-inequality model has similar responses in terms of output and consumption over time. However, the external debt-to-output increases more initially and decreases faster over time comparing to the baseline model. This implies that in the baseline model with inequality the government, despite borrowing less initially, is able to sustain higher debt in the future periods.

Figure 4: Impulse responses to a negative productivity shock



Notes: The graph shows the impulse responses across 30000 paths of the variables for the baseline and no-inequality models. Panel (a), (b), (c), (d) plot the responses of output, consumption, external debt-to-output, and utility inequality to a negative aggregate productivity drop in period 1, respectively.

4 Sensitivity Analysis

This section discusses how the optimal policies quantitatively respond to different model ingredients: government expenditure, aggregate uncertainty, and discount factor. Table 4 compares the moments from the baseline model to other alternative models. The second column reports the moments from the no-government-expenditure model in which $g = 0$. Average external debt-to-output ratio stays the same as the baseline model. This result implies that the amount of exogenous government expenditure does not quantitatively affect the optimal sustainable debt level. Higher government expenditure coincides with higher second moments except for correlation of consumption with output.

The third column of Table 4 considers the deterministic case in which there is no aggregate uncertainty. The deterministic model generates higher level of sustainable debt than the baseline model. An explanation is that uncertainty in the baseline model

Table 4: Role of government expenditure, aggregate uncertainty, and discount factor

	Baseline	No govt. exp. ($g = 0$)	Deterministic ($\sigma_z = 0$)	Lower discount ($\beta = 0.95$)
Avg. external debt/output (%)	17	17	22	12
Std. external debt/output (%)	2.1	0.99	-	1.8
Std. consumption / Std. output	1.2	0.96	-	1.2
Std. net savings/output (%)	1.8	1.5	-	0.62
Correlation with output (%)				
Consumption	95	97	-	99
Net savings/output	31	29	-	-1.1

Notes: The moments are calculated from the model's simulation for 10500 periods and then excluding the first 500 periods. For the second moments, output and consumption series are logged and linear detrended, and net saving and external debt ratio series are linear detrended.

implies a higher precautionary motive than the deterministic case, which leads to the lower overall external debt accumulation. The fourth column describes the moments of the model with lower discount factor ($\beta = 0.95$) than the baseline model. The average external debt-to-output ratio is lower when the discount factor is lower. This is because a higher discount factor implies a lower value of autarky and a longer time period it takes for the economy to reach the borrowing constraints in the baseline economy. Therefore, it allows the government to accumulate and sustain a higher external debt level. In addition, the lower-discount model poorly matches the second moments of net savings in the data.

5 Empirical Evidence

This section aims to assess whether the theoretical prediction of inequality and external debt finds empirical support in a cross-country panel dataset. Proposition 2.2 establishes that there is a positive correlation between pre-tax income inequality and external debt levels. To measure a country's external indebtedness, I use the negative of the net foreign asset-to-GDP ratio from the External Wealth of Nations Database of Lane and Milesi-Ferretti (2018).[21] The database contains data on foreign assets and foreign liabilities. For income inequality, I use pre-tax (market) Gini indices from Standardized World Income Inequality Database (SWIID) of Solt (2019). I focus on the sample of countries

[21] The net foreign asset (NFA) position of a country is the value of the assets that country owns abroad, minus the value of the domestic assets owned by foreigners, adjusted for changes in valuation and exchange rates.

that are often subject to debt crises. Appendix A.3 provides the list of countries in the analysis. Table 5 reports the regression results of net foreign liability-to-GDP on pre-tax Gini index for the time period of 1985 to 2015. Across different specifications, the results show that the correlation between pre-tax income inequality and external debt is positive and statistically significant.

Table 5: Regression analysis of income inequality and external debt

	Net foreign liability-to-GDP (%)			
	(1)	(2)	(3)	(4)
Gini index, pre tax (%)	0.773***	0.472**	4.947***	5.456***
	(0.222)	(0.236)	(0.664)	(0.749)
GDP per capita (log)		-4.972**	-5.781	-1.608
		(1.952)	(4.093)	(7.211)
Real GDP per capita growth (%)		-1.311**	-1.371**	-1.427**
		(0.638)	(0.543)	(0.608)
Current account-to-GDP (%)		-2.061***	-1.090***	-1.100***
		(0.297)	(0.344)	(0.376)
Inflation (%)		0.013***	0.010***	0.010**
		(0.003)	(0.004)	(0.004)
Country fixed effect	No	No	Yes	Yes
Time fixed effect	No	No	No	Yes
No. Countries	30	30	30	30

Note: The table reports the regression coefficients and standard errors in parenthesis of pre-tax Gini index (%) with respect to net foreign liability-to-GDP (%). Control variables are log of GDP per capita, GDP growth (%) and GDP-deflator inflation rates (%). * p<0.1, ** p< 0.05, *** p<0.001. Sources: Lane and Milesi-Ferretti (2018), Solt (2019), and The World Bank (2019).

6 Conclusion

This paper proposes a theory of external debt sustainability that comes from the motive for redistribution of the government. I introduce the government's redistributive concern and distortionary taxation into a sovereign debt framework. I analyze the interaction between distortionary and distributive effect of fiscal policies and the government's lack of commitment. The endogenous borrowing constraints arise from the government's lack of commitment, and become relevant in the long run due to the domestic agents' impatience. Redistribution comes with the cost of tax distortions.

The paper's theoretical contribution is the effect of redistribution on optimal debt sustainability. Given that the government wants to redistribute towards lower skilled lower income households, initial wealth inequality is positively correlated with skill inequality, and conditions on the intratemporal and intertemporal elasticities, it is optimal for the government to sustain positive external debt. Default leading to financial autarky is costly not only because the government cannot use debt to smooth consumption over the business cycles, but also because redistribution is more distortionary and less efficient in financial autarky than in the contract. The distributive cost of default is endogenous and novel to the literature.

The quantitative contribution is showing that the effect of redistribution is quantitatively significant to match observed debt levels in the data. The result contributes to the ongoing literature on endogenous default costs in sovereign debt models. The quantitative analysis shows that by taking into account the redistributive cost of default, the model can quantitatively account for 71% of the long-run average external debt-to-output, while the insurance cost of default alone can only accounts for 12% the long-run average external debt-to-output. The key factor driving these results is the distortionary cost of redistribution. Furthermore, the theory is quantitatively consistent with the cross-country and time-series correlation between income inequality and external debt in the data.

The model has implications on optimal austerity policies in the presence of inequality. Estimations from the model's simulation points out that a negative productivity shock leads to an increase in external debt and a temporary decrease in borrowing taxes, while labor taxes remain unchanged. The average tax-to-income ratio initially decreases for all agents and more for high-income agents, and utility difference decreases. In the future, the government raises average taxes and reduces redistribution to repay debt.

Future extensions of the model can incorporate equilibrium defaults and various types of debt crises.

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Online Appendix

A Data

A.1 Macroeconomic Data Descriptions and Sources

Most data are annual series covering the 1985-2015 period. Some data samples cover the 2002-2015 period.

- Net foreign liability is the negative of net foreign asset (NFA) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Net international investment position is the official international investment position (IIP) from the External Wealth of Nations Database, Lane and Milesi-Ferretti (2018)
- Pre-tax Gini Index the market Gini from the Standardized World Income Inequality Database, Solt (2019).
- GDP per capita is the constant 2010 US Dollar GDP per capita series from World Development Indicator Database, The World Bank (2019)
- GDP growth is the log difference of constant 2010 US Dollar GDP series from World Development Indicator Database, The World Bank (2019)
- Inflation is the annual inflation series measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)
- Real GDP is GDP series in constant local currency units from World Development Indicator Database, The World Bank (2019)
- Real return on German bond is the interest rate on Gernman bond adjusted for inflation measured by the GDP deflator. The interest rate is the long-term interest rate for converenge purposes from the Eurostat Database (2019). These bonds have 10-year maturity and are denominated in Euro.
- Real interest rate is the lending interest rate adjusted for inflation as measured by the GDP deflator from World Development Indicator Database, The World Bank (2019)

- Italy's cross-sectional wage inequality is calculated from the micro-data by Jappelli and Pistaferri (2010) using Surveys of Household Income and Wealth conducted by the Bank of Italy for the period 1980-2006.
- Government consumption is the general government final consumption expenditure series from World Development Indicator Database, The World Bank (2019)
- Private consumption is the households and NPISHs final consumption expenditure series from World Development Indicator Database, The World Bank (2019)

A.2 Italian Household Survey

For the analysis of wage inequality, I use household-level data from the *Survey on Household Income and Wealth* (SHIW), conducted by the Bank of Italy. The SHIW includes cross-sectional and panel data on household's demographics, income, labor supply, consumption, and wealth. Jappelli and Pistaferri (2010) provide details on the survey design descriptions and data quality analysis. I use data for the period 1987 to 2014. In this period, the survey was conducted biennially, except for the period 1995 to 1998 with a 3-year interval.

The sample selection is followed. The original individual sample includes 329,446 units from 1987 to 2014. Given the focus on wage inequality and labor supply across households, I focus on heads of households between the age of 25 and 60. This selection criteria reduces the sample size to 71,621 units. To reduce the impact of outliers, I exclude observations with no hours worked, with negative income, or with hourly wage in the bottom 0.5% of the distribution. The final sample size is 67,176.

A.3 Lists of Countries

Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Greece, Guatemala, Honduras, India, Indonesia, Italy, Ireland, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Portugal, Spain, Sri Lanka, Thailand, Tunisia, Turkey, Venezuela.

B Sovereign Game

In this section, I set up the strategic sovereign game and define the sustainable equilibrium that is characterized by the self-enforcing constraints. All histories are written out explicitly to illustrate the strategic interaction between the government, private agents, and international lenders.

I consider the general environment where the government's policy includes the decision to default on external bond $\{\delta(s^t)\}$, where $\delta \in \{0, 1\}$ and $\delta = 0$ implies default.[22] The government's budget constraint becomes

$$\begin{aligned}&G(s^t) + (1 - \tau^d(s^t))B^d(s^t) + \delta(s^t)B(s^t)\\ \leq&\tau_t^n(s^t)w(s^t)L(s^t) + \sum_{s_{t+1}|s^t} Q^d(s_{t+1}|s^t)B^d(s^{t+1}) + \sum_{s_{t+1}|s^t} Q(s_{t+1}|s^t)B(s^{t+1}) + T(s^t)\end{aligned}$$

The price of international debt takes into account the probability of default is

$$Q(s_{t+1}|s^t) = \frac{\Pr(s_{t+1}|s^t)\delta(s_{t+1}|s^t)}{1 + r^*}$$

As the government cannot commit to any of its policies, one can think that the government, private agents, and international lenders enter in a sovereign game where they determine their actions sequentially. In every period and every history, the state variable for the game is $\left\{B(s^t), \left(b^{i,d}(s^t)\right)_{i\in I}\right\}$. The timing of the actions is as follows.

- Aggregate shock s_t is realized
- Government chooses $z_t^G = \left(\tau^n(s^t), \tau^d(s^t), T(s^t), \delta(s^t), B(s_{t+1}, s^t), B^d(s_{t+1}, s^t)\right) \in \Pi$such that it is consistent with the government budget constraint.
- Agents choose allocation $z_t^{H,i} = \left(c^i(s^t), l^i(s^t), b^{d,i}(s_{t+1}, s^t)\right)$ subject to their budget constraints, the representative firm produce output by choosing $z_t^F = L(s^t)$, and the international lenders choose holdings of government's bonds $z_t^* = B(s_{t+1}, s^t)$.

Define $h^t = \left(h^{t-1}, z_{t-1}^G, \left(z_{t-1}^{H,i}\right)_{i\in I}, z_{t-1}^F, z_t^*, s_t\right) \in H^t$ as the history after shock s_t is realized. Note that the history incorporates the government's policy, allocation and prices. Define $h_p^t = \left(h^t, z_t^G\right) \in H_p^t$ as the history after the government announce its policies at period t. The government strategy is $\sigma_t^G : H^t \to \Pi$. The individual agent's strategy is $\sigma_t^{H,i} : H_p^t \to \mathbb{R}_+^3 \times \mathbb{R}$. The firm has strategy $\sigma_t^F : H_p^t \to \mathbb{R}_+^2$, and the international lenders have strategy $\sigma_t^* : H_p^t \to \mathbb{R}_+$.

Definition B.1 (Sustainable equilibrium)**.** A sustainable equilibrium is $\left(\sigma^G, \sigma^H, \sigma^F, \sigma^*\right)$ such that (i) for all h^t, the policy z_t^G induced by the government strategy maximizes the

[22]Since the paper focuses on characterizing the no-default equilibrium, the set-up from the main text does not explicitly model the default decision and instead takes into account that the government will pay back its foreign debt ($d_t = 1$).

socially weighted utility given λ subject to the government's budget constraint (4) (ii) for all h_p^t, the strategy induced policy $\left\{z_t^G\right\}_{t=0}^{\infty}$, allocation $\left\{z_t^{H,i}, z_t^F, z_t^*\right\}_{t=0}^{\infty}$, and prices $\{Q_t,\}_{t=0}^{\infty}$ constitute a competitive equilibrium with taxes.

The following focuses on characterizing a set of sustainable equilibrium in which deviation triggers autarky, where there is no domestic and foreign borrowing. In this case, the value of deviation includes the autarkic payoff.

By definitions, autarky is a sustainable equilibrium. Given that the domestic agents do not save/invest, the representative firm produces only with labor, and the international creditors do not lend, the government finds it optimal to default on its external debt, set saving and capital taxes such that the after-tax gross returns on domestic bonds and capital are zero, and set the labor tax such that it maximizes the socially weighted utility. Given the government defaulting and fully taxing all returns from domestic savings and capital, international creditors do not want to lend, agents do not save or invest in capital, and output is produced only by labor. Lastly, given that the government will be in autarky in the future, it is optimal in the current period for the government to also follow the autarkic strategies.

Reverting to autarky equilibrium is defined as a sustainable equilibrium of the above game such that following any government's deviation from the promised plans, the economy reverts to autarky. One can characterize the equilibrium as follows.

Proposition B.1 (Reverting to autarky equilibrium)**.** *An allocation and policy* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *can be supported by reverting to autarky equilibrium if and only if (i) given* z^G*, there exist prices* p *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G, p\right\}$ *is a competitive equilibrium with taxes for an open economy, and (ii) for any* t *and any* s^t*, there exists* $\underline{U}(s^t,t)$ *such that* $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ *satisfies the constraint*

$$\sum_{i\in I}\lambda^i\pi^i\sum_{k\geq t, s^t\subseteq s^k}\beta^{k-t}\Pr(s^k|s^t)U^i(c^i(s^k),l^i(s^k))\geq \underline{U}(s^t,t) \tag{12}$$

Proof. Define $\underline{U}(s^t,t)$ as the maximum discounted weighted utility for the agents in period t, history s^t, when the government deviates. At period t and history s^t, the government taxes all domestic wealth ($\tau^d(s^t)=1$) and redistributes equally across agents, and the government defaults on the external debt. In subsequent period $k>t$, the economy reverts to financial autarky where agents do not save in domestic bonds, and the government is excluded from international lending. This economy ensembles a neoclassical growth closed economy that has an initial aggregate state s_t, distortionary taxation on labor, and equal initial wealth across individuals.

Suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an outcome of the reverting to autarky equilibrium. Then by the optimal problems of the government, agents, and foreign lenders, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes the weighted utility of the agents, satisfies government budget constraint and foreign lender's problem at period 0. Thus, $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ is an open-economy tax-distorted competitive equilibrium. For any period t and history h^t, an equilibrium strategy that has the government deviates in period t triggers reverting to autarky in period $k > t$. Such strategy must deliver the weighted value at least as high as the right-hand side of (12). So $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii).

Next, suppose $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies conditions (i) and (ii). Let h^t be any history such that there is no deviation from z^G up until period t and history s^t. Since $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ maximizes agents period-0 weighted utility, it is optimal for the agents if the government's strategy continues the plan from period t and history s^t onward. Consider a deviation plan $\hat{\sigma}^G$at period t that receives $U^d(s_t, t)$ in period t and $U^{aut}(s_t)$ for the subsequent period $k > t$. Because the plan is constructed to maximize the utility in period t, the right-hand side of (12) is the maximum attainable utility under $\hat{\sigma}^G$. Given that $\left\{\left(z^{H,i}\right)_{i\in I}, z^F, z^G\right\}$ satisfies condition (ii), the original no-deviation plan is optimal. $\square$

Proposition B.1 can be extended to the general characterization of sustainable equilibrium, as in Chari and Kehoe (1990).

C Characterizing Sustainable Allocation and Optimal Tax Policies

This section provides details on the characterization of the sustainable allocation and optimal tax policies. Section D will use this analysis to prove the propositions in the main text.[23]

Let μ be the multiplier on the resource constraint, $\pi^i\eta^i$ be the multiplier on the implementability constraint for agent i, and $\beta^t \Pr(s^t)\gamma(s^t)$ be the multiplier on the aggregate debt constraint for period t. Define $\boldsymbol{\eta} = (\eta^i)_{i\in I}$ and rewrite the Larangian of the planning problem with a new pseudo-utility function that incorporates the implementabil-

[23] This analysis is similar to Tran-Xuan (2021) in an economy with aggregate uncertainty.

ity constraints:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^W \frac{C_t^{1-\sigma}}{1-\sigma} - \Phi_L^W \frac{L(s^t)^{1+\nu}}{1+\nu} \right] - \Phi_C^V C_0^{-\sigma} \sum_{i \in I} \pi^i \eta^i \left(b_0^i - T \right)$$

where Φ_C^W, Φ_L^W depend on $\boldsymbol{\varphi}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\eta}$ (see Appendix D.1).

The first-order conditions of the planning problem for any period $t \geq 1$ can be summarized as

$$F_L(L_t, s^t, t) = \frac{\left\{ \Phi_L^W + \Phi_L^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma_k \right\} L_t^{\nu}}{\left\{ \Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma_k \right\} C_t^{-\sigma}} \tag{C.1}$$

and

$$Q(s_{t+1}|s^t) = \beta \Pr(s_{t+1}|s^t) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t+1} \gamma_k}{\Phi_C^W + \Phi_C^P \sum_{k=0, s^k \subseteq s^t}^{t} \gamma_k} \tag{C.2}$$

The optimal tax policies follow

$$\tau_t^n = 1 - \frac{1}{F_L(L_t, s^t, t)} \frac{\Phi_L^V L_t^{\nu}}{\Phi_C^V C_t^{-\sigma}} \tag{C.3}$$

$$\frac{Q_t^d(s_{t+1})}{1 - \tau_{t+1}^d} = \beta \Pr(s_{t+1}|s^t) \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \tag{C.4}$$

D Formulas and Proofs

D.1 Formulas

Given the formulas for ψ_c^i and ψ_l^i in (7), we have the followings:

$$\Phi_C^V = \left[\sum_i \pi^i (\varphi^i)^{1/\sigma} \right]^{\sigma}; \qquad \Phi_L^V = \omega \left[\sum_i \pi^i \left(\varphi^i \right)^{-1/\nu} (\theta^i)^{(1+\nu)/\nu} \right]^{-\nu}$$

$$\Phi_C^P = \Phi_C^V \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_c^i; \qquad \Phi_L^P = \Phi_L^V \sum_{i \in I} \pi^i \frac{\lambda^i}{\varphi^i} \psi_l^i$$

$$\Phi_C^W = \Phi_C^V \sum_{i \in I} \pi^i \psi_c^i \left[\frac{\lambda^i}{\varphi^i} + (1-\sigma) \eta^i \right]; \qquad \Phi_L^W = \Phi_L^V \sum_{i \in I} \pi^i \psi_l^i \left[\frac{\lambda^i}{\varphi^i} + (1+\nu) \eta^i \right]$$

D.2 Proof of Proposition 1.1

Proof. ($\Rightarrow$) Let $\{(C_t, L_t)\}_{t=0}^{\infty}$ be an aggregate allocation of an open economy competitive equilibrium with taxes. Then by definition, $\{(C_t, L_t)\}_{t=0}^{\infty}$ satisfies the aggregate resource constraint. Moreover, given any market weights φ, (C_t, L_t) satisfies

$$(1-\tau_t^n)w_t = \frac{\Phi_L^V L_t^\nu}{\Phi_C^V C_t^{-\sigma}}$$
$$\frac{Q_t^d(s_{t+1})}{1-\tau_t^d} = \beta \Pr(s_{t+1}|s^t)\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$$

Substituting for w_t into the budget constraint (2), and using $c_t^i = \psi_c^i C_t, l_t^i = \psi_l^i L_t$, gives the implementability constraint for each agent. Importantly, choose φ and T such that the individual implementability constraints hold.

$(\Leftarrow)$ Given φ, T and an allocation $\{(C_t, L_t)\}_{t=0}^\infty$ that satisfies the aggregate resource constraint, and individual implementability constraints, construct $\{w_t\}_{t=0}^\infty$ using the firm's first-order condition (3). $\{\tau_t^n\}_{t=0}^\infty$ can be calculated using the intra-temporal condition (8), and choosing $\left\{Q_t^d(s_{t+1}), \tau_t^d\right\}_{t=0}^\infty$ to satisfy the inter-temporal constraint (9).[24] Define $\{q_t\}_{t=0}^\infty$ by $q_t = \Pr(s^t)/(R^*)^t$.

Rewriting the aggregate resource constraint using $F(L) = wL$ gives

$$\begin{aligned}&\sum_{t\geq 0, s^t} q_t \left\{C_t - (1-\tau_t^n)w_t L_t + T_t\right\} \\ &+ \sum_{t\geq 0, s^t} q_t \left[G_t - \tau_t^n w_t L_t - T_t\right] \leq -B_0\end{aligned} \tag{D.1}$$

Aggregating up the agent's budget constraints implies

$$C_t + \sum_{s_{t+1}} Q_t^d(s_{t+1})B_{t+1}^d = (1-\tau_t^n)w_t L_t + (1-\tau_t^d)B_t^d - T_t$$

or

$$C_t - (1-\tau_t^n)w_t L_t + T_t = (1-\tau_t^d)B_t^d - \sum_{s_{t+1}} Q_t^d(s_{t+1})B_{t+1}^d$$

Substituting the last equation into (D.1) gives the government's budget constraint (4). Thus, $\{(C_t, L_t)\}_{t=0}^\infty$ is the aggregate allocation of the constructed competitive equilibrium with government policies. □

[24] There is indeterminacy in setting the price on domestic debt and the domestic savings tax. Any combination of $\{Q_t^d(s_{t+1}), \tau_t^d\}$ that satisfies the intertemporal constraint is part of the competitive equilibrium. Later on, I consider a particular implementation in which $Q^d = Q$, the price of external debt. τ^d then captures the difference between the domestic and external prices.

D.3 Proof of Proposition 2.1

First, I show that the following lemma must hold.

Lemma D.1. *cov* $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ *and cov* $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$

Proof. The first step is to show that for i and j such that $i \neq j$, $\theta^i > \theta^j \iff \varphi^i > \varphi^j$.

Suppose $\theta^i > \theta^j$ and $\varphi^i \leq \varphi^j$, then $\psi_l^i \leq \psi_l^j$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq \frac{\varphi^i}{\varphi^j} \leq 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > 1$, which is a contradiction.

Suppose $\varphi^i > \varphi^j$ and $\theta^i \leq \theta^j$, then $\psi_l^i > \psi_l^j$. By the definitions of ψ_l, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} > \frac{\varphi^i}{\varphi^j} > 1$. However, $\left(\frac{\theta^i}{\theta^j}\right)^{1+\nu} \leq 1$, which is a contradiction.

Next, the individual implementability constraint is

$$\psi_c^i \Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma} - \psi_l^i \Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu} = \Phi_C^V C(s_0)^{-\sigma} \left(a^i(s_0) - T\right)$$

or

$$\psi_c^i = \psi_l^i \frac{\Phi_L^V \sum_{t,s^t} \beta^t \Pr(s^t) L(s^t)^{1+\nu}}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}} + \frac{\Phi_C^V C_0^{-\sigma} \left(b^i(s_0) - T\right)}{\Phi_C^V \sum_{t,s^t} \beta^t \Pr(s^t) C(s^t)^{1-\sigma}}$$

By the definition of ψ_c^i, $\varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j$, and by the assumption, $\boldsymbol{\theta^i > \theta^j \iff b^i(s_0) > b^j(s_0)}$, which implies that $\theta^i > \theta^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

Thus, $\theta^i > \theta^j \iff \varphi^i > \varphi^j \iff \psi_c^i > \psi_c^j \iff \psi_l^i > \psi_l^j$.

In addition, $\boldsymbol{\theta^i > \theta^j \iff \lambda^i < \lambda^j}$, which implies that

$$\psi_c^i > \psi_c^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$
$$\psi_l^i > \psi_l^j \iff \frac{\lambda^i}{\varphi^i} < \frac{\lambda^j}{\varphi^j}$$

Hence, cov $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < 0$ and cov $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$. □

Now I proceed to the main proof of the proposition.

Proof. In financial autarky, there exist a vector of market weights $\boldsymbol{\varphi}^a$, transfer T^a, and multiplier $\boldsymbol{\eta}^a$that satisfies the conditions in Proposition 1.1 such that

$$\underline{U}(z) \equiv \max_{C_t, L_t, \boldsymbol{\varphi}^a, T^a} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \log C_t - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu}\right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a$$
$$s.t. \quad C_t + G = z_t L_t$$
$$z_0 = z$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}, \Phi_L^{W,a}$ follows the formulas in Appendix D.1.

The optimal labor tax in autarky is constant over time and is equal to

$$\tau^{n,a} = 1 - \frac{\hat{\Phi}_L^V \hat{\Phi}_C^W}{\hat{\Phi}_C^V \hat{\Phi}_L^W} == 1 - \frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)}$$

Lemma D.1 shows that cov $\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)$, cov $\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right) < 0$, which implies that

$$\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right) < \mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)$$

or

$$\frac{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] + \sigma \text{cov}\left(\psi_c^i, \frac{\lambda^i}{\varphi^i}\right)}{\mathbb{E}\left[\frac{\lambda^i}{\varphi^i}\right] - \nu \text{cov}\left(\psi_l^i, \frac{\lambda^i}{\varphi^i}\right)} < 1.$$

Thus, $\tau^{n,a} > 0$ □

D.4 Proof of Proposition 2.2

Proof. Suppose, by contradiction that the autarkic allocation $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty$ solves the planning problem given s_0. Then the planning value at period 0 is $V_0^P\left(\{C_t^a, L_t^a, \varphi^a\}_{t=0}^\infty\right) = \underline{U}(z_0)$.

Fix $\epsilon > 0$. Consider the allocation $\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^\infty$ starting at s_0 and s_1 such that $\hat{C}_0 = C_0^a + \frac{\epsilon}{R^*}\mathbb{E}_{s_1|s_0}\left[F_L(L_1^a) - \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}}\right]$, $\hat{L}_0 = L_0^a$, $\hat{C}_1 = C_1^a + \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}}\epsilon$, $\hat{L}_1 = L_1^a + \epsilon$, and for any $t \geq 2$, $\hat{C}_t = C_t^a, \hat{L}_t = L_t^a$.

First, $\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^\infty$ is feasible because

$$\begin{aligned}
&\sum_{t\geq 0} \mathbb{E}_0 \left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&F(\hat{L}_0) - \hat{C}_0 - G_0 + \frac{1}{R^*}\mathbb{E}_{s_1|s_0}\left[F(\hat{L}_1) - \hat{C}_1 - G_1\right] + \sum_{s_1} \mathbb{E}_{s|s_1} \sum_{t\geq 2}^{\infty} \left(\frac{1}{R^*}\right)^t \left[F(\hat{L}_t) - \hat{C}_t - G_t\right] \\
=&\sum_{t\geq 0} \mathbb{E}_0 \left(\frac{1}{R^*}\right)^t \left[F(L_t^a) - C_t^a - G_t\right] - \frac{\epsilon}{R^*}\mathbb{E}_{s_1|s_0}\left[F_L(L_1^a) - \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}}\right] + \frac{1}{R^*}\mathbb{E}_{s_1|s_0}\left(F_L(L_1^a)\epsilon - \frac{\Phi_L^{P,a}(L_1^a)^\nu}{\Phi_C^{P,a}(C_1^a)^{-\sigma}}\right. \\
=&\sum_{t\geq 0} \mathbb{E}_0 \left(\frac{1}{R^*}\right)^t \left[F(L_t^a) - C_t^a - G_t\right] \\
\geq& B_0
\end{aligned}$$

Second, $\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^{\infty}$ is implementable in equilibrium since there exists a $\hat{T}$ such that $\forall i \in I$

$$\sum_{t\geq 0, s^t \in S^t} \beta^t \left[\Phi_C^{V,a} \psi_c^{i,a} \hat{C}_t^{1-\sigma} - \Phi_L^{V,a} \psi_l^{i,a} \hat{L}_t^{1+\nu}\right] \geq \Phi_C^{V,a} \hat{C}_0^{-\sigma} \left(b^i(s^0) - \hat{T}\right)$$

The flow utilities for all period $t \geq 1$ do not change. That is,

$u_1^P(\hat{C}_1, \hat{L}_1, \varphi^a) = \Phi_C^{P,a} \frac{(\hat{C}_1)^{1-\sigma}}{1-\sigma} - \Phi_L^{P,a} \frac{(\hat{L}_1)^{1+\nu}}{1+\nu} = \Phi_C^{P,a} \frac{(C_1^a)^{1-\sigma}}{1-\sigma} + \Phi_C^{P,a} (C_1^a)^{-\sigma} \frac{\Phi_L^{P,a}(L_1^a)^{\nu}}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} \epsilon - \Phi_L^{P,a} \frac{(L_1^a)^{1+\nu}}{1+\nu} - \Phi_L^{P,a} (L_1^a)^{\nu} \epsilon = u_1^p(C_1^a, L_1^a, \varphi^a)$

$u_t^P(\hat{C}_t, \hat{L}_t, \varphi^a) = u_t^p(C_t^a, L_t^a, \varphi^a), \forall t \geq 2$

and the flow utility in period 0 increases. That is,

$$\begin{aligned} u_0^P(\hat{C}_0, \hat{L}_0, \varphi^a) &= \Phi_C^{P,a} \frac{\left(\hat{C}_0\right)^{1-\sigma}}{1-\sigma} - \Phi_L^{P,a} \frac{\left(\hat{L}_0\right)^{1+\nu}}{1+\nu} \\ &= \Phi_C^{P,a} \frac{(C_0^a)^{1-\sigma}}{1-\sigma} + \Phi_C^{P,a} (C_0^a)^{-\sigma} \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a} (L_1^a)^{\nu}}{\Phi_C^{P,a} (C_1^a)^{-\sigma}}\right] - \Phi_L^{P,a} \frac{(L_0^a)^{1+\nu}}{1+\nu} \\ &= u_0^p(C_0^a, L_0^a, \varphi^a) + \Phi_C^{P,a} (C_0^a)^{-\sigma} \frac{\epsilon}{R^*} \mathbb{E}_{s_1|s_0} \left[F_L(L_1^a) - \frac{\Phi_L^{P,a} (L_1^a)^{\nu}}{\Phi_C^{P,a} (C_1^a)^{-\sigma}}\right] \end{aligned}$$

Lemma D.1 implies that $\frac{\Phi_L^{P,a}(L_1^a)^{\nu}}{\Phi_C^{P,a}(C_1^a)^{-\sigma}} < \frac{\Phi_L^{W,a}(L_1^a)^{\nu}}{\Phi_C^{W,a}(C_1^a)^{-\sigma}} = F_L(L_1^a), \forall s_1$, so $u_0^P(\hat{C}_0, \hat{L}_0, \varphi^a) > u_0^p(C_0^a, L_0^a, \varphi^a)$. Thus, $V_0^P\left(\left\{\hat{C}_t, \hat{L}_t, \varphi^a\right\}_{t=0}^{\infty}\right) > V_0^P\left(\{C_t^a, L_t^a, \varphi^a\}_{t=0}^{\infty}\right)$, which contradicts $\{C_t^a, L_t^a, \varphi^a\}_{t=0}^{\infty}$ being the optimal allocation. □

E Quantitative Appendix

This section provides additional details that is implemented in Section 3 .

E.1 Deviation Utility

The deviation utility $\underline{U}(z)$ is calculated as the maximum weighted utility attained from a competitive equilibrium with taxes of a closed economy where the government does not issue both domestic and external debts.

$$\underline{U}(z) \equiv \max_{c_t^i, l_t^i, \tau_t^n, T} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t^i\right)^{1-\sigma}}{1-\sigma} - \omega \frac{\left(\frac{l_t^i}{\theta^i}\right)^{1+\nu}}{1+\nu} \right]$$

$$\begin{aligned}
s.t. \qquad & C_t + G = z_t L_t \\
& c_t^i + \sum_{s_{t+1}} Q_t(s_{t+1}) b_{t+1}^{d,i} = (1-\tau_t^n) z_t l_t^i + b_t^{d,i} - T_t \\
& (1-\tau_t^n) z_t = -\frac{1}{\theta^i} \frac{\omega \left(\frac{l_t^i}{\theta^i}\right)^{\nu}}{\left(c_t^i\right)^{-\sigma}} \\
& Q_t(s_{t+1}) = \beta \Pr(s^{t+1}|s^t) \frac{\left(c_{t+1}^i\right)^{-\sigma}}{\left(c_t^i\right)^{-\sigma}} \\
& b_0^{d,i} = b_0^{d,j}, \ \forall i, j \in I \\
& z_0 = z
\end{aligned}$$

There exist a vector of market weights $\boldsymbol{\varphi}^a$, transfer T^a, and multiplier $\boldsymbol{\eta}^a$that satisfies the conditions in Proposition 1.1 such that

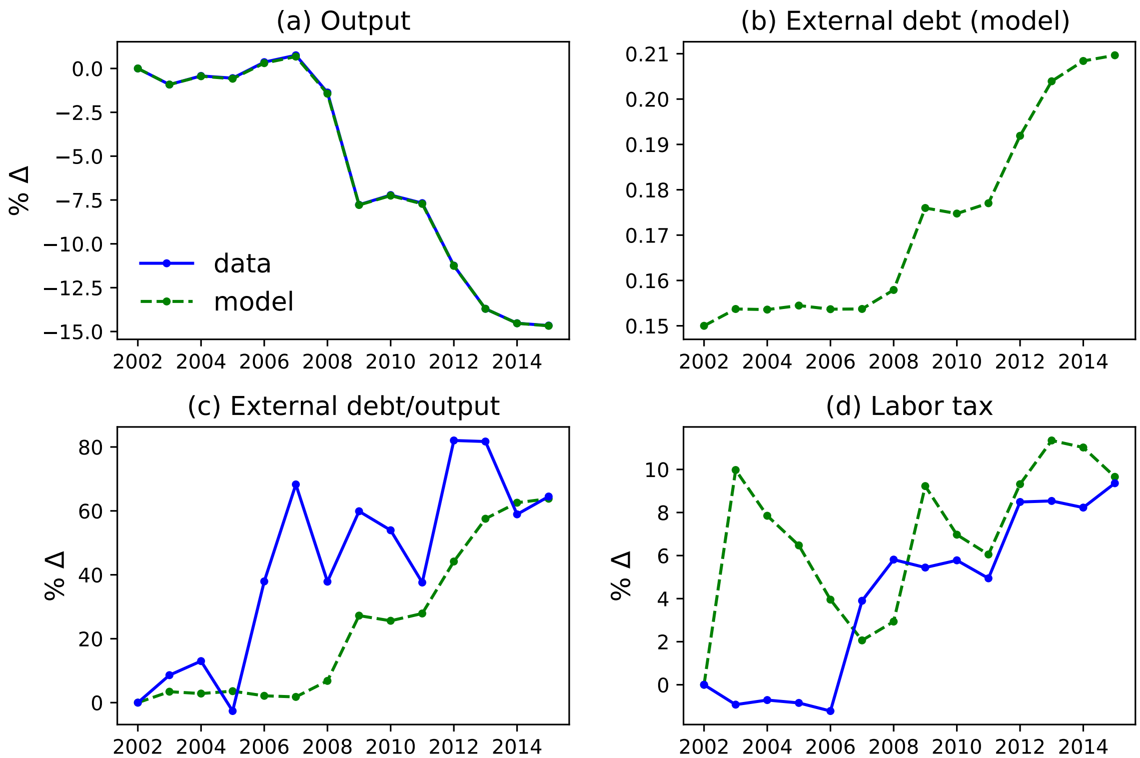
$$\begin{aligned}
\underline{U}(z) \equiv \max_{C_t, L_t, \boldsymbol{\varphi}^a, T^a} & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^{W,a} \log C_t - \Phi_L^{W,a} \frac{L_t^{1+\nu}}{1+\nu} \right] - \Phi_C^{V,a} C_0^{-\sigma} \sum_i \pi^i \eta^{i,a} T^a \\
s.t. \qquad & C_t + G = z_t L_t \\
& z_0 = z
\end{aligned}$$

where $\beta^t \pi^i \eta^{i,a}$ is the Lagrange multiplier on the individual implementability constraint and $\Phi_C^{W,a}$, $\Phi_L^{W,a}$ follows the formulas in Appendix D.1.

E.2 Event Analysis

I conduct an event analysis for Italy in period of 2002 to 2015. I feed into the model a sequence of productivity shock realizations and the initial external debt-to-output ratio such that the model's sequence of outputs matches the sequence of Italian output and the change in external debt-to-output ratio is the same both in the model and the data from 2002 to 2015. I then compare the relative change of external debt-to-output in the data and in the model's simulation over time, given that 2002 is the benchmark year. Figure 5 plots the exercise's results. Panel (a) plots the output paths of the data and the model. Panel (b) plots the time path of external debt in the model. Panel (c) plots external debt-to-output time paths for both the data and the model. Panel (d) plots estimated labor taxes from McDaniel (2007) and the optimal labor taxes from the model. From 2011 to 2015, Italy's output has dropped by 7.6% below trend, accompanying with a 20% increase in external debt-to-output and a 4.2% increase in labor tax. In the model's simulation, the similar drop in output is associated with a 26% increase in external debt-to-output and a 3.5% increase in labor tax.

Figure 5: Italy's Recession: Data and Model



Note: The graph depicts the time paths of output, external debt, and external debt-to-output for the data and the model's simulation. Panel (a) plots the output path. Panel (b) plots the external debt paths of the model. Panel (c) plots external debt-to-output, and panel (d) plots the labor tax. The simulation uses a sequence of productivity shock realization such that the model's output matches the data output for Italy in 2002-2015. The initial external debt level is such that the model's external debt-to-output matches with the starting value in 2002 from the data. The benchmark period is 2002. Data sources: McDaniel (2007), Lane and Milesi-Ferretti (2018), and The World Bank (2019).

E.3 Computational Algorithm

Given the forward-looking borrowing constraints, I implement the recursive formulation developed by Marcet and Marimon (2019).

1. Guess μ and φ. Compute η.

(a) Construct a grid for $\mu_t = (\beta R^*)^t$ for t periods. Construct a grid for Γ
Initial guess for $V(s_t, \mu_t, \Gamma_{t-1}) = \sum_{j\geq 0, s^t \subseteq s^{t+j}} \beta^j \Pr(s^{t+j}) \left[\Phi_C^P \frac{C(s^{t+j})^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^{t+j})^{1+\nu}}{1+\nu}\right]$.

(b) Assume the constraint does not bind in s_t: $\gamma(s_t) = 0$. Solve for the allocation $C(s_t), L(s_t)$ using the first-order conditions

$$\left[\mu_t \Phi_C^W + \Phi_C^V \Gamma_{t-1}\right] C(s_t)^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \Gamma_{t-1}\right] L(s_t)^{\nu} = \mu F_L(s_t)$$

(c) Since $\gamma(s_t) = 0$, compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1})$ (interpolating the expectation), then compute

$$\begin{aligned} A(s_t) &= \sum_{\tau=t}^{\infty} \beta^{\tau-t} \sum_{s^\tau \subseteq s^t} Pr(s^\tau) \left[\Phi_C^P \frac{C(s^\tau)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s^\tau)^{1+\nu}}{1+\nu}\right] \\ &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \Gamma_{t-1}) \end{aligned}$$

(d) Check if $A(s_t) \geq \underline{U}(s_t)$. If it is, proceed to the next step. If not, solve for $C(s_t), L(s_t), \gamma(s_t)$ using these equations

$$\left[\mu_t \Phi_C^W + \Phi_C^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] C(s_t)^{-\sigma} = \mu$$
$$\left[\mu_t \Phi_L^W + \Phi_L^V \left(\Gamma_{t-1} + \gamma(s_t)\right)\right] L(s_t)^{\nu} = \mu F_L(s_t)$$
$$\Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu}$$
$$+\beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) = \underline{U}(s_t)$$

(e) Given $C(s_t), L(s_t), \gamma(s_t)$($\gamma$ can be zero or not), compute a grid at $t+1$ for every possible realization of s_{t+1} given s_t. Compute $V(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \Gamma_{t-1} + \gamma(s_t))$. Update the value function

$$\begin{aligned} V^{n+1}(s_t, \Gamma_{t-1}) &= \Phi_C^P \frac{C(s_t)^{1-\sigma}}{1-\sigma} - \Phi_L^P \frac{L(s_t)^{1+\nu}}{1+\nu} \\ &\quad + \beta \sum_{s_{t+1}} \Pr(s_{t+1}|s_t) V^n\left(s_{t+1}, \mu_{t+1} = \beta R^* \mu_t, \Gamma_t = \beta R^* \left(\Gamma_{t-1} + \gamma(s_t)\right)\right) \end{aligned}$$

2. Compute residuals to find μ and φ

$$
\begin{aligned}
r^{\mu} &= \sum_{t\geq 0} q_t \left[F(L_t) - G_t - C_t\right] - B_0 \\
r^{\varphi} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi_C^V \left(\psi_c^i - \psi_c^j\right) C_t^{1-\sigma} - \Phi_L^V \left(\psi_l^i - \psi_l^j\right) L_t^{1+\nu}\right]
\end{aligned}
$$

3. Find μ and φ such that $r^{\mu} = 0$ and $r^{\varphi} = 0$.